1st Semester
2022-2023

BUSINESS MANAGEMENT

PROBLEMS SET

FINANCIAL CALCULUS

1. Suppose that you have won a prize of 10 000 euros and that you have been asked to choose between the following payment proposals:

   Proposal 1: 5,000 euros today and the rest in 5 annual payments of 1,000 euros, starting to receive one year from now.

   Proposal 2: 3000 euros today and the rest in a year.

   You know that if you want to borrow or invest your money, the relevant interest rate is 4% (annual). Which proposal would you choose?

2. Suppose you are planning to buy a car that costs €10,000 and you have been offered the following offer: either buy it in cash with a 10% discount or give a down payment of €5,000 and pay the rest in 10 months without interest (in equal installments). If the relevant interest rate is 0.5% per month, which alternative do you choose?

3. 8,200 euros are paid to settle two debts, one of 4,000 euros 9 months ago, and the other of 2,000 euros 15 months ago. At what quarterly interest rate does this operation correspond, assuming that capitalization was carried out on a simple basis?

4. Mr. Cifrões invested 3,500 euros for 8 years under a compound interest regime.
   a) Determine the capital accumulated at the end of this period, considering semiannual interest and knowing that the annual interest rate is 10%.
   b) After this period, the accumulated capital will be reimbursed in annual installments in the amount of 1 178 euros. Assuming that the interest rate does not change, calculate the number of years during which Mr. Cifrões will receive this amount.

5. €3,500 were invested in savings certificates. Knowing that after 7 quarters the capital plus the respective interest amounted to 4,116 euros, determine the annual interest rate for that investment.

6. What is the interest rate necessary for the capitalized capital to quintuple, after 25 years, under a compound interest regime?

7. Given an interest rate of 8% per year, what is the value at t=7 of a perpetual flow of payments of €100 starting at t=12

8. "The World Company" paid a dividend of 5 euros today. If the Company increases the dividend by 7% every year and the appropriate discount rate is 11%, what is the price of this Company's shares?

9. (Note: the answer is to be obtained through simulation) You were given the opportunity to buy a security for 12,800 euros. The title certainly pays 2,000 euros at the end of each of the next 10 years. If you buy the bond, what interest rate do you get?

10. Just over a year ago, Renova launched a new cosmetics brand, Dosha. To this end, and after the product definition phase (which will not be detailed here, but which sought, in the definition of a high quality product, to gather references that would give credibility to the brand, surrounding itself with the best specialists, with established credits in the sector , so that there were no reservations regarding the quality of the product – packaging design, laboratory tests, production), it became necessary to invest in the area of communication that would transmit to the consumer "a new language of beauty, based in the concept of self-caring" (ie taking care of oneself) "for women who like themselves the way they are, without fear of the future", as the business director of Renova refers in an interview to Visão magazine in 1999. This investment was of 1 million euros (approximately). Renova was inspired by Shiseido's positioning in the Japanese market, which was an important reference in the company's new investment – in Japan, Shiseido is present in hypermarkets and perfumeries and with very good results. Renova sought to replicate this model. Based on this information, answer the following questions:

a) Consider that the investment (I) in communication made by Renova was to launch Dosha products in large supermarkets. This investment is expected to grow annual sales results (V) at a rate of 5% per year forever from the end of the first year. Assume that the annual interest rate is expected to remain stable at the 10% level. How much do you estimate the results of annual sales have to be at the end of the first year, selling exclusively in large supermarkets, for this investment to be worth making? Use the result you found in the previous question. If you didn't answer the previous question, use the formula given in the theoretical class (on perpetuities with constant growth).

b) Alternatively, Dosha products could have been launched exclusively in perfumeries. In that case, the sales result would start at 30 000 euros at the end of the first year, in contrast to the higher result of the large supermarkets, which would generate 60 000 euros. However, with the same investment in communication made by Renova, the growth rate of these results would be 8%. Which of the two options is actually more interesting for the company?

11. As the manager of a small manufacturing company that usually sells with payment within 30 days, you agree to make an important sale with payment in installments. If the sale amount is 800 thousand euros and the payment is made in 40 monthly installments, how much should the monthly installment be if the relevant interest rate for the company is 7% per year?

12. In a series of 15 annuities, the first five are 200 euros each, the next ones are 400 euros each and the last five are 600 euros each. As the annual interest rate is 12%, calculate its current value and the accumulated value after 15 years.

13. Mr. Oliveira, owner of a small industrial company, needing to purchase a vehicle for his company, went to the dealership of his favorite brand, which advertised credit at a monthly rate of half a percentage point for those who paid, in the act of purchase, half the price of the vehicle. He was interested in the DeLux model, but when he announced to the seller that he intended to pay a monthly installment of €1,000 (maximum), he was told that, with that monthly amount, he would never be able to finish paying for the car. So he was forced to direct his interest to the Economix model, but concluded that at €1,000 a month (plus the initial payment), he would need 5 years to pay for the car, which was not in his plans. He ended up choosing the Baratux model, which costs 20,000 euros, and agreed to pay for it in 2 years.

a) What is the annual interest rate (equivalent) charged by the concessionaire?

b) Given the information given by the seller, what do we know about the cost of the DeLux model?

c) What is the price of the Economix model?

d) What is the monthly installment that Mr. Oliveira agreed for the acquisition of the Baratux model, in order to pay it in 2 years (in addition to the initial payment)?

14. The financial manager of a company has two options to pay suppliers €600,000 in 9 months.

a) The first option is to implement an average period for receiving customers that is shorter than the average period for payment to suppliers in order, without using other financial resources, to pay suppliers with the amount received from customers after being properly capitalized. Under this option, the company would receive 180,000 euros at the end of each of the following three quarters. These amounts can be capitalized (on a compound interest basis) at the quarterly interest rate of 3 percent, which is in effect for the next 9 months.

i) Check if, with this option, the company can meet the payment of 600 000 euros to suppliers in 9 months.

ii) What would be the quarterly interest rate (under compound interest) for, with this option, the company to be able to accumulate 600 000 euros by the end of the ninth month?

b) The second option is to implement the practice of always receiving payment in cash from customers at the end of each month. Next month the first payment will be made, which will grow at a (constant) monthly rate of 0.5 percent.

i) Determine the value of the first payment in order to reach 600 000 euros at the end of the 9 months.

ii) What if the company wants to pay suppliers at the end of the eighth month?

15. A company wants to face the difficult recovery of a long-term (customer) debt of €100,000. You have two options: the first is to sell the debt to a factoring company (which would collect the debt) receiving, today, only 90,164 euros; the second option consists of charging constant semiannual installments of 23,500 euros, starting in 6 months. Assume that the relevant semi-annual interest rate is 1.2 percent for the entire period under review, and that this is a compound interest regime.

a) How many semesters would the company need to collect from customers so that the second option yields as much as the first?

b) Assume now that the company has decided to consider another option for charging. In this third option, you will negotiate with customers the following conditions: receive three constant semi-annual installments of 30,000 euros, the first being received in three months' time. What is the implied discount, d, (i.e. (1-d) * 100 000) that the company is willing to give customers to carry out this plan?

c) For the company, what is the best way to collect debt from customers? Explain.

d) Finally, if the factoring company actually purchased the debt, paying 90,164 euros, and managed to charge customers only in the form of a biannual perpetuity, in the amount of 1,200 euros, starting in 5 years, would have made a profit in this business?

16. Consider that a company has two possible ways to pay its supplier for the merchandise it has just received: in option 1, it provides a down payment of 5 000 euros and will make monthly payments at the beginning of each month, starting immediately, being the value of each monthly fee equal to 2,000 euros in the next eighteen months; in option 2, there will be 6 quarterly payments, starting in a month, and the first amount of which is 200 euros, growing thereafter at a quarterly rate of 20 percent, and a final payment of 25,000 euros. The quarterly interest rate is 2 percent for the first five quarters (i.e. the next 15 months), rising to 3 percent for the remainder of the period. Which is the least expensive option?

17. An entrepreneur is seeking €100,000 loan. Investor "A", who has invested at an annual interest rate of 5.5 percent, proposes to the entrepreneur, in exchange for 100,000 euros, to receive 8 semi-annual installments, starting in one month and always received at the end of the first month of every semester. Alternatively, investor "B", who has an annual remuneration of 6 percent, proposes to lend the 100 000 euros to the entrepreneur upon receipt of 16 quarterly instalments, paid at the beginning of each quarter, the first four being constant and decreasing by thereafter at the quarterly rate of -0.5 percent until the 12th installment, the last 4 installments remaining at that value.

a) State the calculations to be made to obtain the value of the installment required by investor "A" and proceed in a similar way to obtain the value of the first installment for investor "B".

b) If the entrepreneur has a relevant annual interest rate of 8 percent, which of the proposals implies a lower payment during the first year of each loan?