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Space of ideas' expression, modelling and collaboration for the development of imagination, reasoning and learning

Development of Pedagogical Methodology

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Development of Pedagogical Methodology

SUMMARY: This report builds on work done on the ModellingSpace project but also on the educational and psychological literature. A pedagogical methodology is **a set of procedures that a teacher can develop in order to help all students learn**. A methodology is seen as something one cannot receive from others. On the contrary, it is the complex result of **instruction**, personal **experience** and **reflection.**

The report establish a **framework for the procedures**, based on six tenets (Commitment to teaching, to students and to their learning; knowledge of science and mathematics; knowledge of students; knowledge of the art of teaching; science as a way of thinking; and reflection and professional growth) and makes **thirteen proposals for a Modelling Methodology**: (1) make clear goals and plan how concepts and ideas evolve during the activities, anticipating learning difficulties; (2) elicit and verbalize students' conceptions; (3) promote interaction, collaboration, and group cohesion; (4) give prompt feedback; (5) induce self and group formative assessment; (6) proceed from concrete to abstract; (7) verbalize mathematical procedures; (8) promote schematic drawing and writing as "tools-to-think-with"; (9) scaffold the transition from direct computations to algebraic reasoning, from number sense to symbol sense; (10) explore multiple representations; (11) make abstract objects as concrete as possible but spot the differences between the "real thing" and the representation; (12) balance exploratory learning with guided learning; (13) anticipate, check, and revise the coherence of the model and data.

KEYWORDS: Pedagogical/teaching methodology; modelling in science and mathematics; exploratory learning; computers in teaching and learning; collaborative work; teacher education; teacher training.

Contact: Vitor Duarte Teodoro, FCTUNL, Campus da Caparica, 2829-516 Caparica, vdt@mail.fct.unl.pt, tel. +351 917509939.

Main Authors: Vitor Duarte Teodoro
Mônica Mesquita

Contributors: ModellingSpace partners

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What is a Pedagogical Methodology?

It is well know that most people assume that anyone who has been a student can teach ("just remember your best teacher, and do like him...").

This is probably **true... for teaching**, **not for learning**. As a matter of fact, it is now also well known that **learning is not necessarly the outcome of teaching**. For example, in the report *Science for All* (AAAS, 1989), the authors wrote:

> Cognitive research is revealing that even with what is taken to be good instruction, many students, including academically talented ones, understand less than we think they do. With determination, students taking an examination are commonly able to identify what they have been told or what they have read; careful probing, however, often shows that their understanding is limited or distorted, if not altogether wrong.

Teaching can be easy, but helping students learn is surely a not so easy task. Students carry with them many **"learning obstacles"**, ranging from common science misconceptions to epistemological naïve thinking.

A pedagogical methodology is seen, in this report, simply as **a set of procedures that a teacher can develop in order to help all students learn**, not just those who learn almost spontaneously. Note the important verb "**develop**" in this statement: a methodology is not something one can receive from others. It is the complex result of **instruction**, personal **experience** and **reflection**.

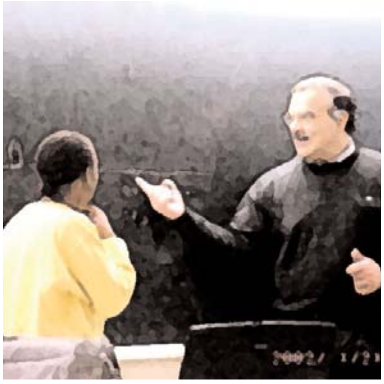
Pedagogical methodologies can vary and change. For example, some decades ago, **reinforcing** (the relationship between the incidence of behaviour, the occurrence of a consequence, and the increased or decreased likelihood of that behaviour occurring in the future) was seen as the essential aspect of a good methodology. Nowadays reinforcement is still considered an important aspect but others are considered more relevant to learning. E.g., **exploring multiple representations** (verbal, graphical, analytical, etc., particularly in science and mathematics) and **concrete experience of abstract concepts** are two of current essential aspects to consider on an effective methodology.

Effective teaching

Effective teaching is seen on the educational and psychological literature as having **multiple components**, such as:

- personal traits of the teacher;
- teacher competencies;
- teaching methods;
- classroom atmosphere;
- teacher decision making-skills;
- students previous knowledge and skills;
- students characteristics.

The interaction between all these factors and the complexity of each make **difficult** (or impossible?) to assert which one is the single **most important factor**.



Some authors, such as Ausubel (1968) *postulate* that students' **previous knowledge** is the single most important factor:

> If I had to reduce all of educational psychology to one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly.

Ausubel also introduced relevant ideas and concepts, such as:

- the distinction **meaningful** and **rote** learning;
- the most **general ideas** of a subject should be presented **first** and them **progressively differentiated**;
- instructional materials must **integrate new** material with **previously** presented information;
- learning materials should be **logically organized** and **potentially meaningful** to learners.
- **anchoring** new concepts into the learner's already existing **cognitive structure** make new concepts recallable.

In this report, we are particularly interested in those factors teachers can manage by themselves. For example, it is not possible for a teacher to have influence on most students' characteristics (e.g., personal characteristics and family background).

Six Tenets of the Pedagogical Methodology

Inspired on the *Standards* developed by the National Board for Professional Teaching Standards, 2001.

Commitment to Teaching, to Students and to Their Learning

Teachers acknowledge and value the individuality and worth of each student, **believe that all students** can learn and **demonstrate** these beliefs in their **practice**.

Knowledge of Science and Mathematics

Teachers **study continuously** to have a **broad and deep knowledge** of the concepts, principles, techniques, and reasoning methods of mathematics and science (and the **connections** between them and with other fields of knowledge), and they use this knowledge to **establish curricular goals** and shape their instruction and assessment.

Knowledge of Students

Teachers **know** and **care** about their students, know **how they learn and develop**, understand the impact of home life and cultural background, and use this knowledge to guide their curricular and instructional decisions.

Knowledge of the Art of Teaching

Teachers have an **extensive base of pedagogical knowledge** to stimulate, motivate and facilitate student learning, using a wide range of formats and procedures to create environments in which students are **active learners**, show willingness to take intellectual **risks**, develop **confidence** and **self-esteem**, and **value knowledge**.

Science as a Way of Thinking

Teachers develop students' abilities to **reason** and think **alone** or **with support** from others, to investigate and explore patterns, to discover structures and establish relationships, to formulate and solve problems, to justify and communicate their conclusions, and to **question and extend** those conclusions.

Reflection and Professional Growth

Teachers **reflect** on what and how they teach and **collaborate** with others to strengthen the **learning community**.

Commitment to Teaching, to Students and to Their Learning

Teachers acknowledge and value the individuality and worth of each student, **believe that all students** can learn and **demonstrate** these beliefs in their **practice**.

In recent years, international studies such as the Trends in International Mathematics and Science Study (TIMSS, http://timss.bc.edu) and the Programme for International Student Assessment (PISA, http://www.pisa.oecd.org) have shown that **most countries face complex problems with student learning** in Science and Mathematics. These studies are being used by governments and schools to promote changes in teaching and learning, not only in Europe but also in many other countries, including the US (see, e.g., http://nces.ed.gov/timss).

Science and Mathematics in schools has always been considered a **difficult subject** by most people, in probably all countries. And it is... particularly when teachers assume that only "the best" students can learn it. The famous **Pygmalion Effect** is a uniquely human phenomenon: a persistently held belief becomes a reality.

At least since late 1960s, the mastery learning and the formative assessment movements and, later, research-based teaching has shown that **most if not all students can learn**, **more concretely or more formally**, the habits of mind, the concepts and the ideas of science and mathematics.

Theories, like Howard Gardner's **Theory of Multiple Intelligences** (1983), recognize that all human beings have different intelligences, connected to core operations (e.g., **logical-mathematical**, connected to number, categorization, and relations; **spatial**, connected to accurate mental visualization, mental transformation of images). Different human beings have different degrees of each intelligence, but **all have some degree of all intelligences**.

Learning how to make models, in science in mathematics, can be done by **all students**, with **different degrees of success**. For example, modelling with tables, as shown below, is easily grasped by all, but modelling with differential equations can only be done by formal thinkers, with a long training path.

Knowledge of Science and Mathematics

Teachers **study continuously** to have a **broad and deep knowledge** of the concepts, principles, techniques, and reasoning methods of mathematics and science (and the **connections** between them and with other fields of knowledge), and they use this knowledge to **establish curricular goals** and shape their instruction and assessment.

Teachers can be defined as **professional learners**. What they learnt in the University is a very small fraction of what they need to learn during their professional life, particularly in subjects like science and mathematics, which are in the front run of intellectual and technological change and progress.

For example, for a teacher that finished his or her professional training in early 1980s, the following is **completely or partially new**: cellular phones, personal computers, universal networks, graphical based software, digital media, genetic manipulation, many materials used in fabrics and sports, etc. These developments where only possible due to science and technology and most are now **part of the everyday life** of students.

It can be said that **foundational knowledge** stays more or less the same for decades. That's probably true. The current scientific paradigms (in the Kuhnian sense, Kuhn, 1962)–quantum mechanics, relativity, plate tectonics, big-bang theory, evolution, genetics, etc., were developed between 1900 and 1980. But who can learn deeply even a very specific subject in the University? And students, particularly the young ones, are always eager for specific knowledge–only a teacher who is always a learner can give "**food for thought"** to his students.

Experienced teacher's supervisors know that those teachers who do not regularly study new and old things tend to have problems appreciating student learning difficulties. That can be easily understood: if someone repeats many times what he teaches, it becomes trivial and completely familiar to him. But **if he is always studying, learning difficulties are constantly present** and he can understand of how difficult it can be for students to learn something they are not familiar with.

Knowledge of Students

Teachers **know** and **care** about their students, know **how they learn and develop**, understand the impact of home life and cultural background, and use this knowledge to guide their curricular and instructional decisions.

Traditional teachings methodologies were based on teaching the same, in the same conditions, with the same approach to all students. On the second half on the 20th century, **"teaching the same to all"** was discarded as a feasible methodology due to multiple factors (generalization of secondary studies, results of educational and psychological research, multiculturalism in schools, etc.).

The now dominant current practices recognize **each student as a different learner**, with different personal knowledge and skills. A good metaphor (suggested by Bruner, 1960) for the students **learning path is a spiral line**. Different students can be at **different places on the spiral line**, on each class. The spiral form suggests that learning progress is not linear and it happens with cycles and steps forward.

According to Bruner, the spiral learning approach can be characterised by a *continuum* with three reference levels: enactive, iconic and symbolic (these levels were inspired on the developmental psychology of Jean Piaget). On the **enactive** level, the child manipulate materials **directly**. On the **iconic** level, deals with **mental images** of objects but does not manipulate them directly. On the **symbolic** level, can manipulate symbols and no longer needs mental images or objects.

To understand how students learn, Bruner also proposed an important principle: **cognitive processes precede perceptions** rather than the other way around. The relevance of this principle was reinforced by science and mathematics constructivist authors. For example, Driver (1983), on her seminal book *The Pupil as Scientist*, wrote:

> 'Looking at' is not a passive recording of an image like a photograph being produced by a camera, but it is an active process in which the observer is checking his perception against his expectations (pp. 11-12).

Knowledge of the Art of Teaching

Teachers have an **extensive base of pedagogical knowledge** to stimulate, motivate and facilitate student learning, using a wide range of formats and procedures to create environments in which students are **active learners**, show willingness to take intellectual **risks**, develop **confidence** and **self-esteem**, and **value knowledge**.

Active or practical learning methods are not a panacea for learning. Practical activities don't mean anything if students don't have **anchoring concepts** on their **cognitive structures**. The **balance** between practical and discovery methods is probably the most important issue a teacher must manage when planning and promoting learning activities. As Driver (1983) pointed out:

> If we wish children to develop and understanding of the conventional concepts and principles of science, more is required than simply providing practical experiences. The theoretical models and scientific conventions will not be 'discovered' by children through their practical work. They need to be presented. Guidance is then needed to help children assimilate their practical experiences into what is possibly a new way of thinking about them (p. 9).

Quoting a famous proverb, («I do and I understand»), Driver changed it to better illustrate how relevant students' knowledge structures are: «I do and I am even more confused».

Papert (1980), a pioneer of computers in education, wrote about the importance of students being **"computer off"**, reflecting and discussing ideas and procedures. Technological devices, such as computers, can reinforce the "button syndrome" (*users tend to press the maximum number of buttons on the shortest time...*) if practical learning activities are done for their own sake, without previous preparation, reflection and discussion.

In a simple short statement, the art of teaching is the **art of balancing "minds-on" activities with "hands-on" activities**, giving students sufficient time to **internalise concepts** and build a **coherent cognitive structure**.

Science as a Way of Thinking

Teachers develop students' abilities to **reason** and think **alone** or **with support** from others, to investigate and explore patterns, to discover structures and establish relationships, to formulate and solve problems, to justify and communicate their conclusions, and to **question and extend** those conclusions.

Science is not *just* about facts. It is best described as a **"way of thinking" about the world**, as the Nobel winner Leon Lederman wrote (1998).

The fist amazing idea of science as a way of thinking is that the **"world is understandable"** (as Einstein pointed out), i.e., the world is accessible to human thought. Humanity took a long time to adopt this point of view. We know now that our understanding of the world is limited–science makes and test models about how the world works, but models are not the "real thing" (see, e.g., Giere, 1989)–but the essence of Einstein's idea is a breakthrough of scientific thought.

A second fundamental way of thinking in science can be described as **informed scepticism**. Popper (1989) has shown that any statement can only be a scientific statement if it "can be tested by systematic attempts to refute them". Contrary to common sense (including students' common sense!), scientific ideas are not dogmas and they are always subject to test and refutation. This doesn't mean that it is easy to accept a new scientific view: for example, Planck wrote, in his autobiography (1950), how difficult it is for most scientists.

Before formal schooling was invented, family, religion and apprenticeship were the available modes to transmit and share ways of thinking, values, attitudes, and skills from one generation to the next. They still have today a relevant place, together with media and peers. But science requires a way of thinking that cannot be transmitted by these modes and need a more formal setting to develop. For example, **careful controlled observation**, based on theory and expectations, is a typical process/way of thinking of science not used in everyday contexts. The same is true for most other scientific processes.

Reflection and Professional Growth

Teachers **reflect** on what and how they teach and **collaborate** with others to strengthen the **learning community**.

A traditional **classroom** is an **isolated place** for the teacher: he is "alone"... with his students. **Working together** with other teachers, using team teaching or just preparing worksheets, is something **relatively new** in most schools and it is not a current practice in many.

Seeing themselves as partners with other teachers is an essential issue, particularly when the use of technology tools is embedded in the curriculum. Teachers have many opportunities and motivations to collaborate, learn, and work together with computer tools, either because these tools facilitate interaction (e.g., email and chat) or because **technological difficulties** cannot most of the time be solved by only one teacher.

Through **reflection and collaboration**, a teacher can develop the art and science of good teaching practice. Reflection requires thoughtful and careful reporting and analysis of teaching practice, philosophy, and experience. Understanding why an activity or practice was productive or non-productive in the classroom is a key element in professional development.

Teacher training has been a recurrent topic of educational research and educational policy. But the concept of "teacher training" is a controversial one because "training" is a narrow concept if one envisions teaching as much more than using a repertoire of teaching techniques. **"Professional development"** or **"professional growth"**, particularly for those who are certified teachers, is a much more inclusive concept to name what is desirable for the teaching profession.

Schools that have a technology embedded curriculum need to share a vision of themselves as **collaborative learning communities** and **learning organizations** (i.e., a group of people who are continually enhancing their capabilities to create what they want to create, Senge, 2000). The pace of change in society and in technology doesn't offer any opportunity for a less demanding vision.

Thirteen Proposals for a Modelling Methodology, with Illustrations

As discussed above, a pedagogical methodology is seen, in this report, as **a set of procedures that a teacher can develop in order to help all students learn**.

A methodology is something a teacher **develops**, based on his or her own experience and knowledge and on proposals made by others (scientists, peers, teachers' educators, etc.). A methodology is, then, a complex result of **instruction**, personal **experience** and **reflection**.

The following thirteen proposals highlight relevant procedures to help teachers build a coherent methodology. Most proposals are illustrated with specific Modelling Space examples in the following pages.

1. Make **clear goals** and **plan** how concepts and ideas **evolve** during the activities, **anticipating** learning difficulties.
2. **Elicit** and **verbalize** students' conceptions.
3. Promote **interaction**, **collaboration**, and group cohesion.
4. Give prompt **feedback.**
5. Induce self and group **formative assessment**.
6. Proceed from **concrete to abstract**.
7. **Verbalize** mathematical procedures.
8. Promote schematic **drawing** and **writing** as **"tools-to-think-with"**.
9. **Scaffold** the transition from **direct computations** to **algebraic reasoning**, from **number** sense to **symbol** sense.
10. Explore **multiple representations**.
11. Make **abstract objects** as **concrete** as possible but spot the differences between the "real thing" and the representation.
12. **Balance** discovery and exploratory learning with guided learning.
13. **Anticipate**, **check**, and **revise** the coherence of models and data.

1

Make clear goals and plan how concepts and ideas evolve during the activities, anticipating learning difficulties

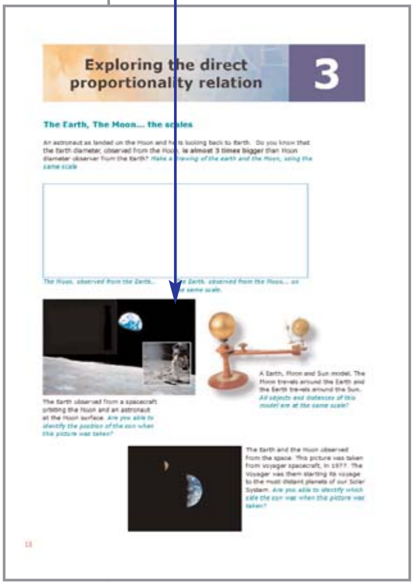
Establishing goals and objectives is a common practice in education since late 1950s, after the publication of Bloom's Taxonomy of Educational Objectives (1956). Objectives and goals offer easily understandable **guidelines for systematic planning and evaluation**, covering the **multiple cognitive processes** (and not only the lower mental processes, such as memorization). Goals must put a bigger emphasis on processes rather than on content matter.

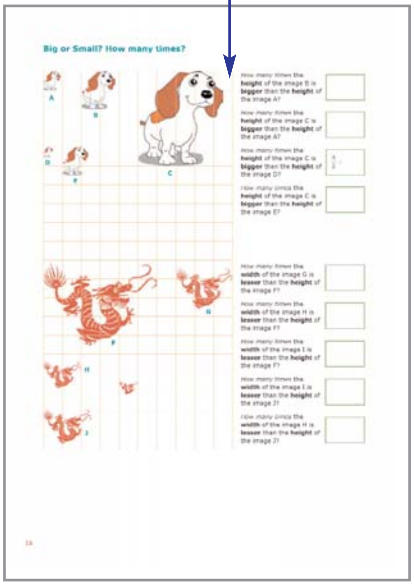
For example, most students have problems understanding **ratios** (e.g., "an object is five times larger than other"). Then, an activity on the direct proportionality relation can be **preceded** by a familiar context, as far as possible, where learners can use ratios. The goal is direct proportionality but before reaching this goal, understanding ratios is a fundamental step. Most learning difficulties can and should be anticipated, giving students less demanding activities, in order to help them make smooth transitions, from familiar contexts to more formal ones.

The goal is a “trip to the Moon”... and some elementary mathematics about proportionality (direct and inverse).

Images and sizes. Using grids, students discuss how many times the height of an image is bigger that other.

This discussion is an essential pre-requisite to understand proportionaly.





2

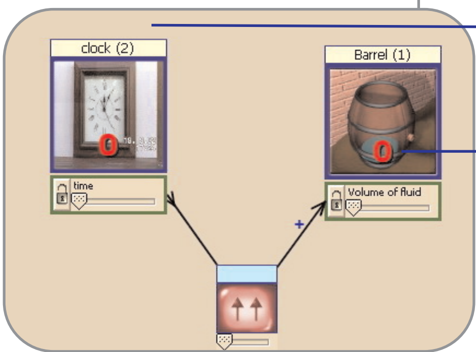
Elicit and verbalize students' conceptions

Eliciting and **verbalizing** is a powerful didactic tool. Teachers must give many opportunities to **students talk about what they see, do, and think**. Typical teacher encouragements can be:

- What are you seeing on the screen (or on the paper)?
- Can you describe what you have done?
- What do you think it will happen?
- What do you think of this?
- What does this word/statement mean to you?
- What are the limits of ...?
- What does this image mean to you?
- What is the relevant information you can gather from this ...?

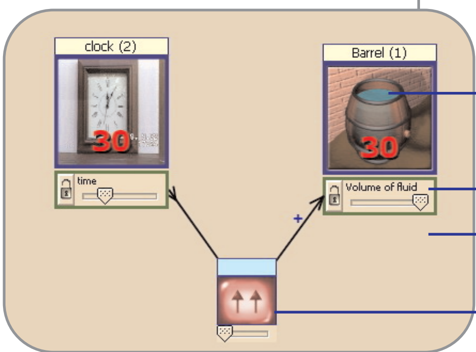
It is possible to include these type of eliciting activities as part of written worksheets but a **balance** must be made be between "**asking**" and "**recording**".

Some examples of eliciting questions:



What happens when time starts? What does it mean to say "time starts running"?

What does "0" means in this image?



How does the height of the water increases as time runs?

What is the maximum value of this variable or entity?

What will happen if time continues to run?

What do you "read" on these two arrows?

Can you plan and make a table of values from this animation?

3

Promote interaction, collaboration, and group cohesion

Most if not all modelling activities that can be done with Modelling Space should be made by **groups** of 2 to 4 students. Teachers can give students the chance to choose their partners, or can put mixed ability students in the same group.

A typical activity starts "computer-off": students read and discuss the goal of the activity, as stated on the worksheet. Teachers supervise students reading and make clear how important is to understand what is the goal of the activity. Students should also be encouraged to **"browse" through all the pages** of the worksheet to have a global idea of its content and purpose, actions and records required, and to have a feeling of the time required to complete it. Certain activities can only be done on more than one class period.

Worksheets should encourage group interaction. E.g., instead of asking "what **do you** think of..." it can say "what **does your group** think of...".

For younger students, it can be interesting to allow students to choose a **name for their group.** Students usually choose "strong and original" names, which helps to create group cohesion and identity. It can also be useful to **assign roles in each group** (e.g., keyboard manager, worksheet record coordinator, spokesperson for the group). These roles can rotate by group members in different activities.

Modelling Space allows collaborative activities, locally and remotely. Collaborative environments demand extra-effort from the teacher to supervise the discussions among the students, as well as their work on the shared or the individual workspaces, but it usually pay-off since **students become more aware of their own reasoning** and **teachers know explicitly how students reason**. Three main strategies can be used: **"tutoring" between groups** (one group explains the other how to make a model); **"coaching" between groups** (one group observes the work done by the other and intervenes to give advices and make suggestions); **"confrontation" between groups** (both groups discuss the work done by the other and both intervene to support actions). In certain cases, activity logs can provide teachers with rich information about students reasoning and knowledge construction.

4

Give prompt feedback

Feedback is the **information** provided to a learner or to a group concerning the correctness, appropriateness, or accuracy of actions. Feedback occurs only **after** an action, is **observable** and **describes the effects** of the action. Feedback is, essentially, **information about performance**.

Feedback is pervasive in teacher-class interactions but it less common on an individual basis since most classroom activities involves the class as whole.

Feedback is generally considered a **critical component** in effective teaching and learning. The original behaviourist view of feedback as a reinforcer automatically linked to responses has been changed to new views and practices with the raise of cognitive theories of learning. Feedback is now viewed as **more linked with learner's cognitive processes** rather then solely with answers produced by learners. Typical instances of feedback involves students describing their thought processes when solving a problem, reading a text aloud and discussing it, managing the computer, listen to the thought process described by teachers, and then compare what they did to what the teacher did.

When using ModellingSpace, teachers have many opportunities to give feedback to students, both on a individual basis and on a group basis. The pedagogical question is how to give prompt feedback. In order to succeed in giving prompt and meaningful feedback, **teachers must be proactive**. The pattern of proactive teacher behaviour includes:

- **walking around** all groups and sitting with each group if necessary;
- **reading aloud** to the group what students have recorded on worksheets and what they have done on the screen and discuss it;
- **asking questions** that can provoke thinking and verbalization.
- **writing comments**, suggestions, and questions **directly on students' worksheets**.
- **making schematic drawings** that describe ideas and processes and **discuss** them with students.

5

Induce self and group formative assessment

Students' views on assessment are still more related to summative assessment (related to knowledge and skills certification) than formative assessment, a concept that arose in late 1960s (Scriven, 1967).

Formative assessment is, essentially, a **way of learning**, and should be part of all learning activities. It provides feedback for improvement of learning and instruction and **occurs throughout the learning process**.

The following are characteristics of formative assessment:

- it is **detailed** and provide **specific information** for **improvement** rather than single composite scores in the form of marks or grades;
- it is **nonthreatening** to the student and to the group so as not to stimulate defensiveness and rejection but they are **conscious** that they are being evaluated;
- it is **timeliness**;
- **value judgments** are explicit and available to the student, in context.

In order to promote formative assessment during Modelling Space activities teachers should:

- **include questions** like "write down your group predictions" on worksheets;
- give preferably **indirect hints** about students' work;
- promote **group interaction** to assess students' answers (e.g., group B analyze the answers of group A);
- show **correctly filled worksheets** to give students reference standards on how to answer certain types of questions;
- **make value judgments explicit** to students and groups, accompanied by reflection hints (e.g.: "This is correct, can you explain why?". "Something is still missing in this in order to be correct. Can you find what is missing?").

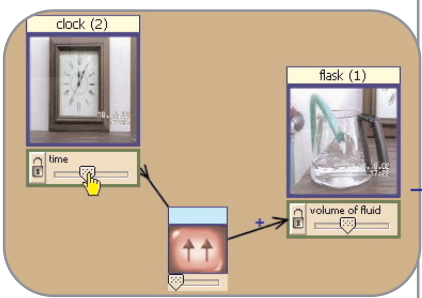
6

Proceed from concrete to abstract

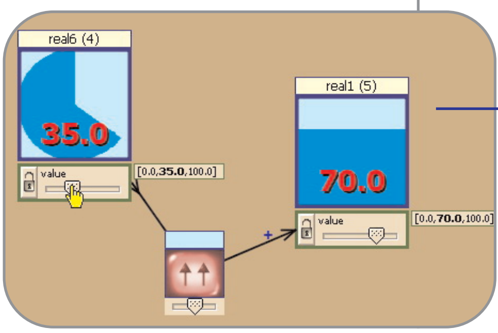
Science and mathematics **is about formal models of objects**, not about the objects themselves (Giere, 1989). This is a subtle and difficult distinction to novices. **Novices' knowledge** usually consists mostly of the **objects and features** explicitly presented in specific situations. For example, most novices classify introductory physics problems as, e.g., inclined plane problems or projectile problems. Their solution procedures are usually syntactic and specific, translating problem statements directly into "formulas", that should have an almost immediately "solution". In contrast, **experts' knowledge** tend to represent problems by the **relevant implicit** physics concepts, quantities and principles, usually not mentioned directly in the problem description (Chi et al. 1981).

As multiple authors have shown, in child development there is a **shift** from **verbatim representations** to **gist representations**, a **shift** from **concrete to abstract**. Introductory Modelling Space activities should be designed to scaffold this shift. For example:

- whenever adequate, **start with concrete entities**, like "car" or "clock";
- discuss what are the **properties** of each entity **relevant** to a certain problem situation;
- **define operationally** the relevant properties when analyzing a problem;
- discuss how can properties be **measured**, i.e., expressed by **numbers** that represent **quantities**.



A "concrete" model, made with "objects" (images have a high degree of perceptual fidelity). Some questions to discuss: What are the relevant properties of the "clock" and the "recipient" one is interested to model the situation? How can the properties be operationalized and mesuared?



A more formal model of the same problem... and of many similar problems.

7

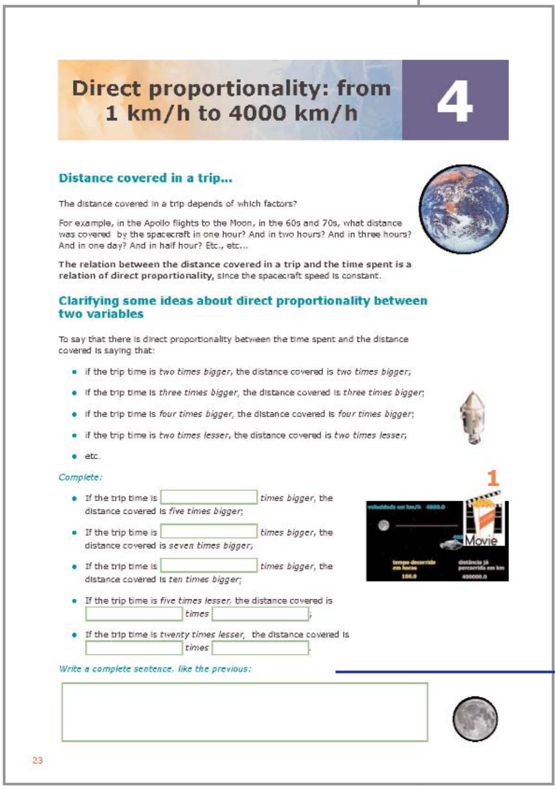
Verbalize mathematical procedures

Students can learn to articulate reasoning by presenting their thinking to themselves, to their peers and to adults.

Verbalization is a **powerful heuristics** students can use to learn and to become conscious of how and what they have learnt, growing into more self-sufficient learners. **Self-sufficient learners** (Miller and Brewster, 1992) are those who know what can affect their cognition, know how to control their own cognitive endeavours, and believe that cognitive effort results in academic success. This heuristic encourages students to talk (and write) to themselves and to their peers as they engage in learning and problem-solving processes. The types of verbalizations include:

- **analyzing the task** and the goals to be achieved (e.g., "what do I have to do?");
- formulating a **plan** of action (e.g., "how should I do it?");
- **describing** by words mathematical relationships (e.g., "if this increases *n* times, this decreases *n* times");
- **evaluating the effectiveness** of each of the steps in the plan (e.g., "how am I doing?");
- **giving themselves positive feedback** for succeeding with each step (e.g., "that's fine work.");
- dealing with obstacles, employing corrective actions (e.g., "that's not completely right.").

Teachers should model and display these steps and the verbalizations in learning situations, in order to provide students with information about why the verbalizations are necessary and how effectively they work.



Direct proportionality: from 1 km/h to 4000 km/h

4

Distance covered in a trip...

The distance covered in a trip depends of which factors?

For example, in the Apollo flights to the Moon, in the 60s and 70s, what distance was covered by the spacecraft in one hour? And in two hours? And in three hours? And in one day? And in half hour? Etc., etc...

The relation between the distance covered in a trip and the time spent is a relation of direct proportionality, since the spacecraft speed is constant.

Clarifying some ideas about direct proportionality between two variables

To say that there is direct proportionality between the time spent and the distance covered is saying that:

- if the trip time is *two times bigger*, the distance covered is *two times bigger*;
- if the trip time is *three times bigger*, the distance covered is *three times bigger*;
- if the trip time is *four times bigger*, the distance covered is *four times bigger*;
- if the trip time is *two times lesser*, the distance covered is *two times lesser*;
- etc.

Complete:

- If the trip time is ____ *times bigger*, the distance covered is *five times bigger*;
- If the trip time is ____ *times bigger*, the distance covered is *seven times bigger*;
- If the trip time is ____ *times bigger*, the distance covered is *ten times bigger*;
- If the trip time is *five times lesser*, the distance covered is ____ *times* ____;
- If the trip time is *twenty times lesser*, the distance covered is ____ *times* ____.

Write a complete sentence, like the previous:

1

Movie

23

This introductory page is only about promoting verbalization of linear relationships...

8

Promote schematic drawing and writing as "tools-to-think-with"

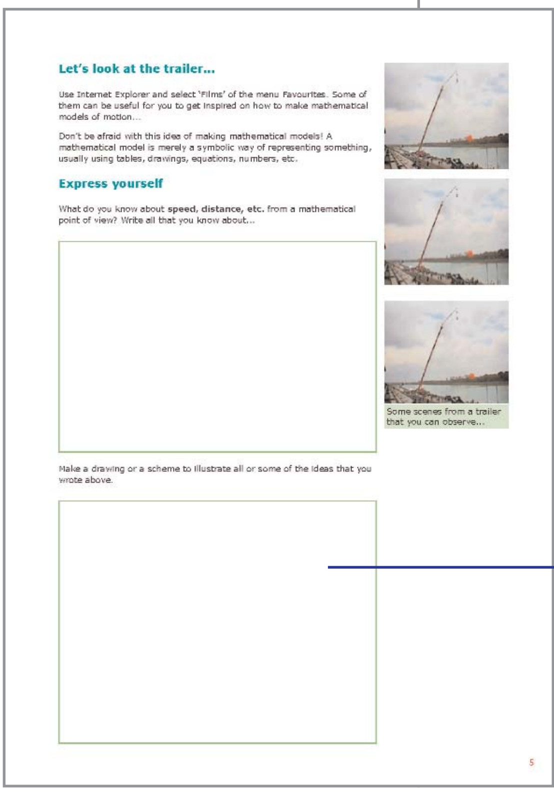
Young students' abilities to talk and listen are usually more advanced than their abilities to read and write. Therefore, teachers must provide experiences that allow varied forms of written communication as a natural component of modelling activities.

Recording and **communication** should always be **explicit activities**. Students can make **different types of records**, ranging from words, simple sentences, simple drawings or more elaborated records, like concept maps, careful schematic drawings or even full reports.

Drawing and writing are usually preferable to talk. In fact, drawing and writing demands **more time and consciousness** than talk and can be more easily **revised** by students.

Drawing and writing **usually starts with thinking-aloud**, an activity that must be common to teachers and students alike. Students can learn to think aloud if the teacher usually illustrates that procedure and shows how it can be used to make records and re-examine reasoning.

A good record is effective to oneself and to other students. For example, the record below is a clear restatement of what's on the text and can easily be understood by any reader. Students become better at listening, paraphrasing, questioning, and interpreting others' ideas when they can record them, using their own words and drawings. With time, their records become more precise.



Let's look at the trailer...

Use Internet Explorer and select 'Films' of the menu Favourites. Some of them can be useful for you to get inspired on how to make mathematical models of motion...

Don't be afraid with this idea of making mathematical models! A mathematical model is merely a symbolic way of representing something, usually using tables, drawings, equations, numbers, etc.

Express yourself

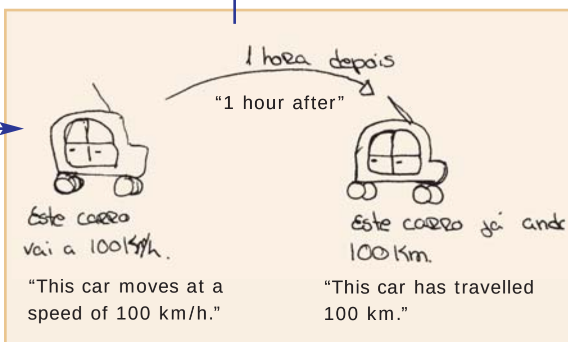
What do you know about **speed, distance, etc.** from a mathematical point of view? Write all that you know about...

Some scenes from a trailer that you can observe...

Make a drawing or a scheme to illustrate all or some of the ideas that you wrote above.

5

A drawing made by a student to describe what happens to the distance travelled by a car in one hour.



9

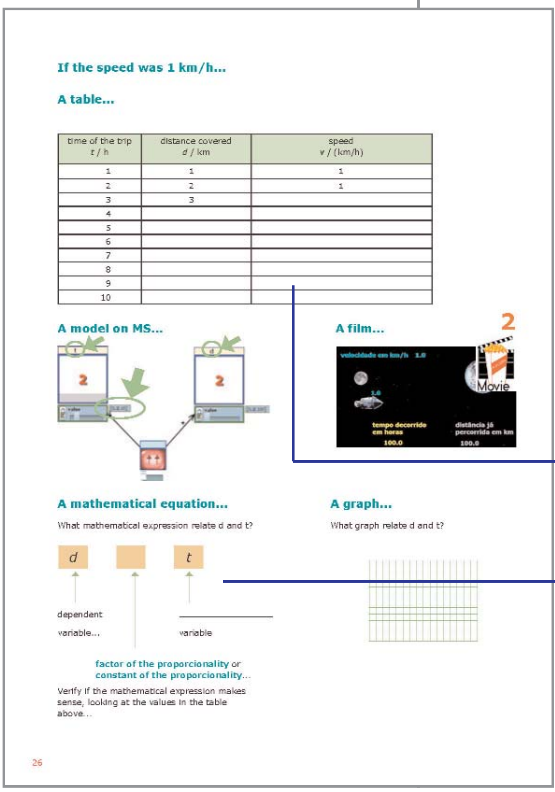
Scaffold the transition from direct computations to algebraic reasoning, from number sense to symbol sense

Different authors (e.g., Kaput, 2001) have argued that developing **algebraic reasoning** since elementary levels is **critical** for mathematics teaching and learning. It is necessary to reconceptualize the nature of algebra and algebraic thinking and how it relates to arithmetic. The traditional but **artificial separation of arithmetic** (where numbers are treated as "real objects" that one can manipulate) **and algebra** (where symbols are treated as quantities and "real objects"–symbol sense) makes more difficult for students to learn algebra and denies students of powerful mathematical ideas.

Familiarity takes a long time to develop, and algebraic thinking is no exception. But one must be careful because the idea is not to push the current typical high school algebra curriculum to more elementary grades. The goal is only to develop algebraic reasoning, not skilled use of common algebra procedures.

The two central algebraic procedures are (1) **making generalizations** and (2) **using symbols** to represent mathematical quantities and solve problems. A typical generalization is that multiplying a number by a fraction of 1 results on a smaller number and a common symbolic procedure is representing that "quantity *a* is the double of quantity *b*" by "$a = 2 \times b$".

Modelling Space activities must smoothly support this transition from arithmetic reasoning (e.g., making direct computations on tables of values) to algebraic reasoning (e.g., transforming a table of values of two variables into an equation).



If the speed was 1 km/h...

A table...

time of the trip t / h	distance covered d / km	speed v / (km/h)
1	1	1
2	2	1
3	3	
4		
5		
6		
7		
8		
9		
10		

A model on MS...

A film...

A mathematical equation...

What mathematical expression relate d and t?

factor of the proporcionality or constant of the proporcionality...

Verify if the mathematical expression makes sense, looking at the values in the table above...

A graph...

What graph relate d and t?

26

This table requires "number sense"...

This mathematical equation illustrates the transition from direct computations, as on the table, to symbolic algebraic representation.

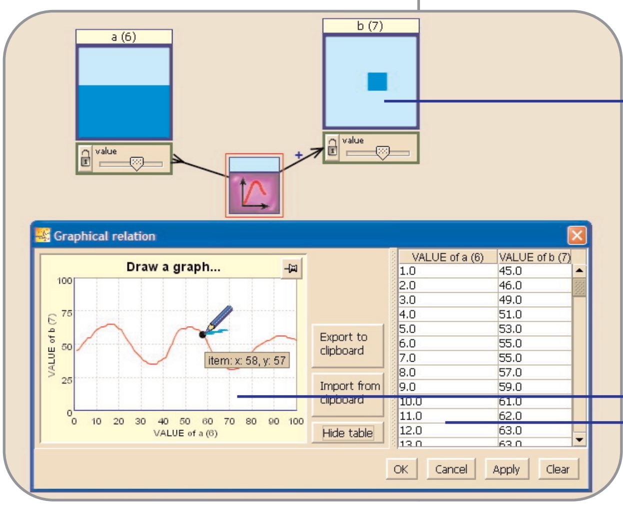
10

Explore multiple representations

Multiple representations (Perkins, Schwartz, West, & Wiske, 1995) are **alternative coordinated views of a phenomenon**, model or process in a certain cognitive domain, such as a graph, an equation, and a table.

The concept of multiple representations has been a recurrent concept in exploratory software design for science and mathematics, at least since the publication of *Making sense of the future* (Harvard Educational Technology Center, 1988). In this position paper, the authors argue about how computers can make a difference in learning environments: they stress the fact that computers can easily present simultaneously representations of the same formal object, such as a function (e.g., the analytical expression, a table of values, and a graph). Multiple representations, emphasizing different aspects of the same idea and affording different sort of analyses, are now a "taken for granted" issue in most educational software for science and mathematics.

Modelling Space activities should encourage teachers to use multiple representations, particularly of **simple mathematical relationships, including those that cannot be expressed by algebraic expressions**. For example, the model below shows an oscillatory relation between variable *b* (on the right) and variable *a* (on the left). The relation is established as a "graph" but a "table" is also available. Running the model, the small square that represents the value of *b* oscillates back and forth. Another representation, the "verbal" one, should be asked on the worksheet...



The small square, representing the value of variable *b*, oscillates back and forth as variable *a* increases.

The oscillatory relation between variable *b* and variable *a* was established using the mouse but it can also be done introducing values on the table.

11

Make abstract objects as concrete as possible but spot the differences between the "real thing" and the representation

In the early beginning of computers in education, Papert (1980) wrote:

> Stated most simply, my conjecture is that the computer can concretize (and personalize) the formal. Seen in this light, it is not just another powerful educational tool. It is unique in providing us with the means for addressing what Piaget and many others see as the obstacle which is overcome in the passage from child to adult thinking. I believe that it can allow us to shift the boundary separating concrete and formal. Knowledge that is accessible only through formal processes can now be approached concretely. And the real magic comes from the fact that this knowledge includes those elements one needs to become a formal thinker (p. 21).

Reification (using formal objects as real objects to think with) is at the realm of learning science and mathematics (Roitman, 1998). Hebenstreit (1987) coined a term that seems essential to understand how computers can help in the reification of knowledge. For Hebenstreit, computers allow the user to manipulate a new type of objects, objects that he calls **concrete-abstract objects**. Concrete in the sense that they can be manipulated on the screen and react as "real objects" and abstract because they can be only physical or mathematical constructs such as magnitudes, equations, fields, etc.

Modelling Space entities and relations are, in most contexts, concrete-abstract objects, sometimes **multiple levels of perceptual fidelity**. Using abstract objects concretely, as in Modelling Space, can be a powerful didactic heuristics. But there also pitfalls, since a significative number of students tend to consider representations as the "real thing" (Justi & Gilbert, 2002).

12

Balance discovery and exploratory learning with guided learning

The inquiry curricula, associated with **discovery and exploratory learning**, were inspired by Piaget and Bruner's ideas, notably the *The act of discovery* paper (1961), where Bruner argues that discovery learning is superior to learning based on expository modes.

A very different position has been assumed by other cognitive theorists such as e.g. Ausubel (1963, 2000). Ausubel's most common critique of discovery learning is that although it can be effective in certain specific situations, for the most part it is cumbersome and overly time-consuming. Discovery learning also demands from the teacher a greater contextualization in order to have a better chance of retention than rote memorization of a procedure. Accordingly to Ausubel (2000), **most meaningful learning is associated with reception learning**, not with discovery learning. In opposition to Bruner, Ausubel argues that discovery learning can also be "rote in nature because it does not conform to the conditions of meaningful learning" (p. 5) and that **meaningful reception learning is an active process**, not a passive one, and requires cognitive analysis in order to define which aspects of existing cognitive structure are most relevant to the new potentially meaningful material.

Balancing exploratory and guided learning is a fundamental issue in the use of computer tools like modelling software. Teachers should always bear in mind that **learners cannot explore what they don't know already!**

The balance between exploratory learning and direct instruction must be managed by curriculum developers and by teachers. This balance can be supported via two means:

1. the **student activity sheets**;
2. **teachers' strategies** during the sessions.

Activity sheets play a crucial role for this balance. When someone try to ask questions in a written way, it is difficult to balance it , and it is difficult to avoid saying something that finally, guide students directly. Often a question that is presented later, reveal something that students have to discover previously. For this reason, one need to **gradual delivery short and separate sheets**.

13

Anticipate, check, and revise the coherence of models and data

Models are abstract constructions of human mind. They cannot be “true” in the sense that they are “like the real thing”. **Models only represent some features** of the “real thing”. There are two important words in the previous statement: ***represent*** and ***some***. *Both words* establish the epistemological status of models. These status must be **explicitly discussed and learnt** by students.

At all levels of modelling, from planning a real trip to the Moon to learning simple models of direct and inverse proportions, a fundamental issue is the anticipation of the coherence of the model and data.

For example, **a student can start making a model** of the trip to the Moon **anticipating** that **time of travel grows with distance and decreases with speed...** The data for this statement comes from common sense experience but it can also come from other sources, such as graphs of experimental results.

Anticipation is absolutely fundamental to check if the model makes sense, once it is done. If the student doesn’t anticipate what he will get with the model, he is more ready to accept anything... and that happens frequently, usually based on a mislead view that “the computer is allways right”.

A possible **sequence for modelling activities** is:

1. **discuss** and **describe the features** of what is going to be modelled;
2. **select** what **properties** will be used on the model;
3. **anticipate** possible **relations between the properties** (if possible, **get real data** on these relations);
4. **analyse** and **discuss** the relations and **make a first model**, guided by a worksheet, by the teacher, or by group discussion;
5. **run** the model and **compare** the output with the predictions;
6. make “what-if” investigations, **discuss** and **revise**, as necessary.

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