Proceedings of the 25th IASTED International Conference
MODELLING, IDENTIFICATION, AND CONTROL
February 6-8, 2006, Lanzarote, Canary Islands, Spain

LINEAR AND NONLINEAR CONTROL OF MUSCULOSKELETAL SYSTEMS

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ABSTRACT
The methods of nonlinear systems and control theory were applied to a simple musculoskeletal system with one joint and two muscles in this paper. For this purpose, a nonlinear input-affine state-space model has been developed for this system with a flexor and an extensor muscle taking into account the nonlinear muscle and movement dynamics. Two types of controllers were designed: a linear pole-placement servo designed for the locally linearized model, and a nonlinear controller based on asymptotic output tracking method. It is expected and forward that during a movement with a large range of motion the nonlinearities of the system will be dominant and the nonlinear methods will outperform the linear ones. The output reference tracking properties of the controllers have been investigated by simulation and it has been found that the nonlinear controller gives more accurate tracking with no overshoots than the linear one.

KEY WORDS
Non-Linear Control, Linear Control, Biomedical Modelling, Arm Motion

1 Introduction

Even the simplest musculoskeletal system exhibits strongly nonlinear dynamic behavior that calls for applying the results of nonlinear systems and control theory. At the same time, the proper control of musculoskeletal systems is important for designing and controlling artificial limbs, muscle prosthesis and in neuro-physiological and Functional Electrical Stimulation (FES) investigations. Therefore, the aim of this work is to examine the different kind of controls of such a strongly nonlinear system, as a musculoskeletal system.

A number of papers can be found in the literature that deal with the design and evaluation of various controllers for musculoskeletal systems. In the papers found in the area of biomechanics and movement control generally static (e.g. [1]) or dynamic (e.g. [7, 13]) optimization is applied in a feed-forward manner for controller design. In these studies the movement is controlled based on the minimization of some key performance variables, such as minimization of net force, net activation, fatigue etc. When the controller design is based on optimization, the nonlinear behavior is usually generally taken into account but the computing cost of the design is very high and these methods are generally not robust to the disturbances.

Thelen et al. [19] proposed the so called computed muscle control method to make faster optimization by applying feedback. Applying this method they computed the state of the muscles and then the value of excitation therefrom. De Sapio et al. [5] developed the task-level control of human motion, that is similar to control of Thelen et. al. [19]. It is suitable for control of goal directed motion and posture control at the same time.

In the area of posture control the controller is generally designed by using engineering methods (feedback controllers) based on locally linearized system models. Therefore, these controllers are not able to take into account the nonlinear dynamics of the system but their design and operation are computationally less demanding than that of the ones based on dynamic optimization. The feedback nature of these controllers can provide stable response to disturbances and external interactions. Khang and Zajac [10], for example, designed an LQ-like (Linear Quadratic) controller for maintaining the standing posture with FES applying the dynamic equations linearized around the standing posture. Kooij et al. [11] also developed an LQ-like controller to the linearized system equations for maintaining the standing posture by integrating all available sensory information. Peterson and Chizeck [14] developed a LQ control for a loaded agonist-antagonist muscle pair. Their control law translates the position and velocity of external load into an optimal applied force for position tracking. Riener and Fuhr [15] recommended a control strategy which accounted for voluntary upper body effort during the control of standing up, but did not require the estimation of hand reactions.

During the last decades it has been shown in various fields of engineering (electrical, mechanical, process engineering etc.) that the application of traditional PID-control and even the methods of modern control theory based on locally linearized state-space models often do not give satisfactory results when applied to systems exhibiting significant nonlinearities. Paralelly, many theoretically well-founded results have appeared in the field of analysis and control of dynamic systems given in nonlinear state-space form (see e.g. [12], [9] and [16]). The main drawbacks of some nonlinear control techniques (especially of the

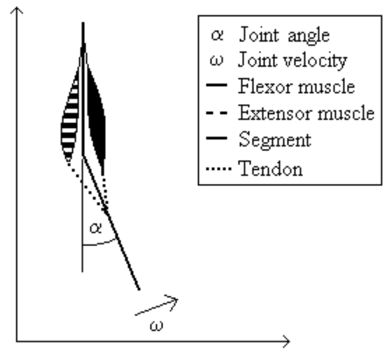

Figure 1. Simple musculoskeletal system

methods based on exact or input/output linearization) are that they can be quite sensitive to model uncertainties and in many cases they require the exact measurement of the whole state vector. Luckily, the rapidly improving quantity and quality of measurements and actuators allows us firstly to build high-fidelity nonlinear models suitable for nonlinear controller design [6], and secondly to implement the computed (often complex) feedback laws. Thus the application of nonlinear control theory has the potential to provide a fast and efficient controller that is able to take into account the nonlinear dynamics of the musculoskeletal system at the same time.

2 Methods and Materials

2.1 Model

A model developed by Csercsik *et al.* [4] was applied for the controller design. This musculoskeletal model is a simple 1-degree-of-freedom one-joint system with a nonlinear flexor and a nonlinear extensor muscle (see figure 1) suitable for nonlinear systems analysis and control. Parameters and variables belonging to flexor muscle have got $_f$ sub-index while parameters and variables belonging to extensor muscle have got $_e$ sub-index in this paper. The dynamic model of a one-joint system with two muscles contained the nonlinear dynamics of movement [21] and the nonlinear dynamics of the muscle contraction [8, 17, 20]. The nonlinearities of movement dynamics originate from the gravitational effects and the geometry of the model, while nonlinearities of muscle dynamics originate from the nonlinear properties of the muscles, such as force-length-velocity relation, activation dynamics, passive force and tendon nonlinear dynamics. Inputs of the system were the normalized exciting signals of each muscle, i.e. $u = [u_f,\ u_e]^T$, where the elements should obey the constraints: $0 \leq u_f \leq 1;\ 0 \leq u_e \leq 1$. The performance output was the joint angle, i.e. $y = \alpha$, while its time derivative, the joint angle velocity ω has served as a secondary output.

The dynamic segments of the musculoskeletal system were supposed to be rigid. The nonlinear equation (1) below describes the movement dynamics, i.e. how torques act on the moving musculoskeletal system:

$$\frac{d\omega}{dt} = \frac{1}{\Theta + ml_{com}^2}\left(M + ml_{com}\cos\left(\alpha - \frac{\pi}{2}\right)g\right) \quad (1)$$

where α [rad] is the joint angle, ω [rad/s] is the angle velocity, Θ [kgm^2] is the moment of inertia defined to the mass-center point of the bone, m [kg] is the mass of the moving segment, l_{com} [m] is the distance between the moving segment's center of mass point and the joint axis, M [Nm] is the resulting joint torque, and g [m/s^2] is the gravitational acceleration. The correction term $\frac{\pi}{2}$ means that the direction of the first, fix segment was vertical as it can be seen in figure 1.

The current length of the muscle is computed form the current joint angle using the cosine theorem. The muscle is divided into two parts: from the origin to the closest point to the joint, and from the closest point to the joint to the insertion. The length of both muscle parts are computed using the cosine theorem. We consider that the shortest distance between muscle and joint is the moment arm of the muscle.

The crucial component of the model was the part that generates the exerting muscle forces. A muscle model was then converted into a state-space form where the following eight state variables were applied:
- Joint angle: α
- Joint angle velocity: ω
- Muscle activation states (2 pieces): q_χ
- Tendon lengths (2 pieces): l_T^χ
- Tendon extracting velocities (2 pieces): v_T^χ

where $_\chi =_{f,e}$ refers to the type of muscle, and thus

$$x = [q_f,\ q_e,\ \alpha,\ \omega,\ l_T^f,\ l_T^e,\ v_T^f,\ v_T^e]$$

The torque is computed by the equation

$$M = F_f d_f - F_e d_e \quad (2)$$

where F_f [N] and F_e [N] are the forces of flexor and extensor muscle, respectively, acting on the joint, and d_f, d_e [m] are the moment arms of the flexor and extensor muscle, respectively. The force of the flexor muscle is computed by the equation:

$$F_f = F_f^{max}\left(FL\left(l_f^{CE}\right)FV\left(v_f^{CE}\right)q_f + F_{PE,f}^{max}F_f^{PE}\right) \quad (3)$$

where F_f^{max} [N] is the maximal force of flexor muscle, $FL(l_f^{CE})$ is a normalized nonlinear force-length relationship [17], $FV(v_f^{CE})$ is the normalized, nonlinear force-velocity relation, q_f is the activation state of the flexor muscle, F_f^{PE} [N] is the passive force generated by the flexor muscle and $F_{PE,f}^{max}$ is a constant showing the ratio between the maximal passive force and maximal isometric force of the flexor muscle.

The activation dynamics is described by a first order differential equation [20] in case of both muscles:

$$\frac{dq_f(t)}{dt} = -\left(\frac{1}{\tau_{act}^f}\left(\beta_f + (1-\beta_f)\,u_f(t)\right)\right) q_f(t) + \frac{1}{\tau_{act}^f} u_f(t) \quad (4)$$

where $u_f(t)$ [1] is the flexor muscle exciting signal, τ_{act}^f [s] is the time constant of muscle activation when the muscle is fully excited ($u_f(t)=1$), $\tau_{deact}^f = \frac{\tau_{act}^f}{\beta_f}$ is the time constant of muscle deactivation when the muscle is fully deactivated ($u_f(t)=0$) and β_f is a constant.

The force-length relation $FL(l_f^{CE})$ is the same as used by [17], i.e.

$$FL\left(l_{CE,f}\right) = c_f\left(\frac{l_{CE,f}}{l_{CE,f}^{opt}}\right)^2 - 2c_f\left(\frac{l_{CE,f}}{l_{CE,f}^{opt}}\right) + c_f + 1 \quad (5)$$

where $l_{CE,f}$ [m] is the current length of the flexor muscle, $l_{CE,f}^{opt}$ [m] is the optimal flexor muscle length, c_f is a constant.

The nonlinear mechanical properties of muscles were described based on [17, 20] but the static nonlinear functions were approximated to fit them better to the control purpose. The original force-velocity function $FV(v^{CE})$ described by Hill [8] and extended by van Soest and Bobbert [17] was not continuously differentiable, so to avoid computational problems, we used an approximating smooth function to meet the requirements of nonlinear analysis (the parameters of the function were found by parameter fitting):

$$F_v(v_{CE}) = -\frac{3}{2}\arctan\left(\frac{9}{5}v_{CE} - \frac{9}{25}\right)\pi^{-1} + \frac{167}{200} \quad (6)$$

where v^{CE} [m/s] is the contraction velocity.

The tendons are modeled with a second order differential equation for the sake of simplicity:

$$\frac{l_T^f}{dt} = v_T^f \quad (7)$$

$$\frac{dv_T^f}{dt} = -\frac{k_T^f(l_T^f - l_{T,f}^{slack}) + s_T^f v_T^f - F_f}{z_T^f} \quad (8)$$

where l_T^f [m] is the length of the tendon of flexor muscle, v_T^f [m/s] is the elongation velocity of tendon of flexor muscle, $l_{T,f}^{slack}$ [m] is the slack length of the tendon of flexor muscle, and k_T^f, s_T^f and z_T^f are constants. The k_T^f is determined such that the elongation of tendon is 4% [18] when the muscle force is equal to its maximal isometric force and s_T^f and z_T^f have to be much smaller. Similar functions were used for the extensor muscle.

The values of the parameters were chosen to respect the real operating conditions of the elbow joint and elbow muscle. Therefore, the joint angle should fulfill $0 \le \alpha < \pi$.

2.2 The applied controller design techniques

Because the applied nonlinear control methods require a single output single input (SISO) model, the model was divided into two parts. One of them contained an active flexor muscle and an inactive extensor muscle, while the other contained an inactive flexor muscle and an active extensor muscle, thus each part became a SISO model. Control input was designed separately for these two parts and switching between them was controlled by appropriate rules.

This separation means that only one of the muscles can get excitation signal at a given time. This assumption is close to reality, because during a lot of movements either the agonist or the antagonist muscle is active [21].

2.2.1 Pole-placement servo controller design

The purpose of this simple controller design method is to achieve asymptotic stability and constant reference tracking in the closed loop system. Let us assume that the state equations of the linearized equations of the open-loop system are given by

$$\dot{x} = Ax + Bu \quad (9)$$

$$y = Cx, \quad (10)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the manipulable input and $y \in \mathbb{R}$ is the output to be controlled. Let us extend the linearized model in the following way

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}\begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0^{n\times 1} \\ 1 \end{bmatrix} y_R \quad (11)$$

where $y_R \in \mathbb{R}$ is the reference.

If the extended model (11) is controllable, then we can place the poles of the closed loop system arbitrarily using a full state feedback

$$u = -K\begin{bmatrix} x \\ z \end{bmatrix}, \quad K = [K_x \quad K_z] \quad (12)$$

to achieve the required dynamic response and constant reference tracking. We note that K in (12) can also be computed as a result of an optimal (e.g. linear quadratic) design problem, but in our particular application the pole-placement method is easier to use to avoid undesirable overshoots in the controlled output.

2.2.2 Asymptotic output tracking of SISO nonlinear systems

Consider a nonlinear single-input single-output dynamical system in the following input-affine state-space form

$$\dot{x} = \mathbf{f}(x) + \mathbf{g}(x)u \quad (13)$$

$$y = \mathbf{h}(x) \quad (14)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output, $\mathbf{f}$ and $\mathbf{g}$ are smooth $\mathbb{R}^n$-valued mappings, $\mathbf{f}(x^0) = 0$ and $\mathbf{h}$ is a smooth $\mathbb{R}$-valued mapping.

It is said that (13) has relative degree r at the equilibrium x^0 if $L_\mathbf{g} L_\mathbf{f}^k \mathbf{h}(x) = 0$ for all x in a neighborhood of x^0 and all $k < r - 1$, and $L_\mathbf{g} L_\mathbf{f}^{r-1} \mathbf{h}(x^0) \neq 0$, where $L_\mathbf{f}\mathbf{h}$ denotes the Lie-derivative of $\mathbf{h}$ along $\mathbf{f}$.

Consider a SISO nonlinear system of the form (13) having relative degree r at its equilibrium. Let us denote the (not necessarily constant) reference to be tracked by $y_R(t)$ and define the tracking error e as

$$e(t) = y(t) - y_R(t) \tag{15}$$

It is shown e.g. in [9] that by using the following static nonlinear feedback law

$$\begin{aligned} u \quad = \quad & \frac{1}{L_\mathbf{g} L_\mathbf{f}^{r-1}\mathbf{h}(x)} \Big(-L_\mathbf{f}^r \mathbf{h}(x) + y_R^{(r)} \\ & - \sum_{i=1}^{r} c_{i-1} (L_\mathbf{f}^{(i-1)} \mathbf{h}(x) - y_R^{(i-1)}) \Big) \end{aligned} \tag{16}$$

the tracking error has the following linear dynamics

$$e^{(r)} + c_{r-1} e^{(r-1)} + \cdots + c_1 e^{(1)} + c_0 e = 0. \tag{17}$$

This means that by choosing the design parameters $c_0, \ldots, c_{r-1}$, the dynamics of the tracking error can be shaped appropriately (naturally, keeping the input constraints in mind, too).

2.3 Controller design, tuning and comparison

For the design of the pole-placement servo controller, the nonlinear model of the musculoskeletal system was linearized around the current reference joint angle.

The poles of the closed loop system were placed on the real axis on the left half-plane to avoid sinusoids in the system response and they were chosen so that the control input should satisfy the physical constraints (i.e. its value is between zero and one). The method of pole-placement was the following. If a pole of the open-loop system had a nonnegative real part, then it was replaced to approximately -5 on the real axis. A pole with a negative real part and a nonzero imaginary part was simply projected onto the real axis (i.e. the imaginary part was set to zero).

It can be shown [4] that the musculoskeletal described in section 2.1 has relative degree 3 in the whole operating region for both inputs. To compute the nonlinear feedback law (16), the following parameter values were used: $c_0 = 500000$, $c_1 = 100000$ and $c_2 = 5000$. These values were determined experimentally to obtain good performance while satisfying the input constraints.

The two control approaches were tested using a piecewise constant reference signal which can be seen in fig. 2. The performance comparison of the controllers were based

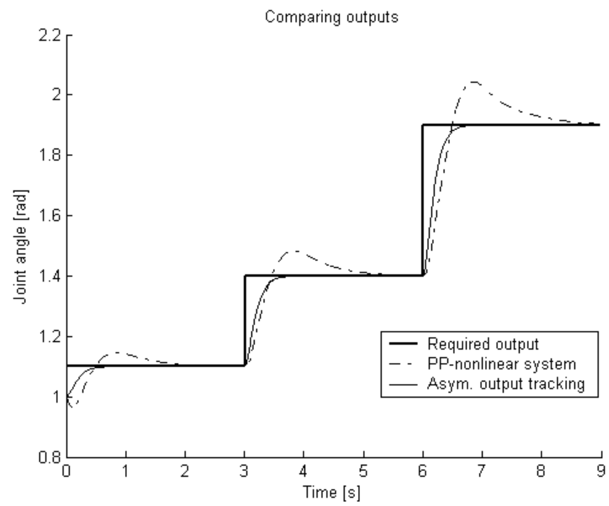

Figure 2. Controller outputs for a piecewise constant reference

Table 1. Input and error norms

	Input norm	**Error norm**
Pole-placement servo	2.0508	2.948
Asy. output tracking	1.9536	2.0561

on the computation of the 2-norm of the tracking error and that of the necessary control input. Note that the simple 2-norm of the input is only loosely correlated to the actual energy used by the muscles (see e.g. [21]). The settling time and maximal overshoots were also examined during the controller evaluation.

The nonlinear controller was additionally tested by a piecewise linear (not constant) reference signal for examining the possibilities of the tracking of more complex reference signals.

3 Results

3.1 Tracking of a piecewise constant reference

Responses to the piecewise constant reference are shown in fig. 2. It is visible that the settling time of the pole-placement servo controller is higher than that of the nonlinear one, while there are no overshoots in case of asymptotic output tracking. The inputs generated by the controllers can be seen in fig. 3. As it can be seen, all the inputs are within the prescribed range.

The input and error norms (see in table 1) in case of asymptotic output tracking are lower than in the case of the pole-placement servo. Therefore we can conclude that asymptotic output tracking gives a better performance with lower input energy.

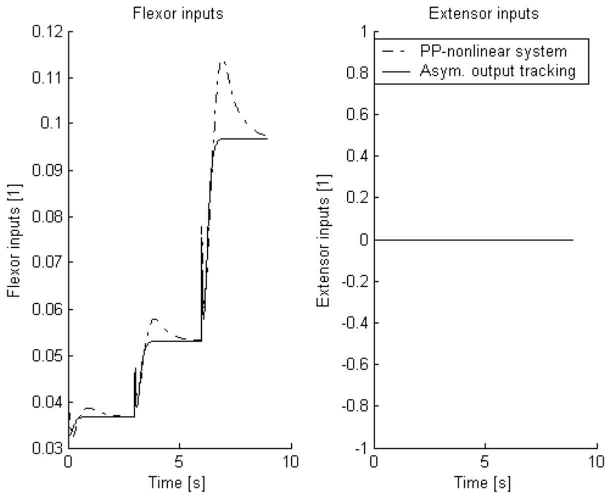

Figure 3. Inputs generated by the controllers in the case of piecewise constant reference tracking

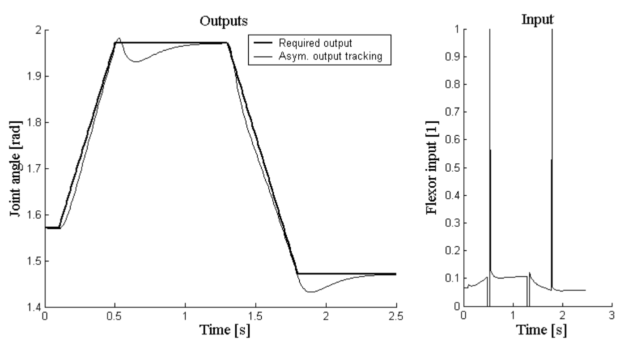

Figure 4. Tracking of piecewise linear output and its inputs generated by the asymptotic output tracking controller

3.2 Tracking of a piecewise linear function

Applying asymptotic output tracking nonlinear control the musculoskeletal system can be controlled to track the required piecewise linear joint angle function. This task has not feasible for the pole-placement controller. The results are shown in fig. 4. The slopes of the piecewise linear sections are 0, 1 and -1 rad/s. Greatest tracking errors occurred when the required output became constant. Overshooting is less than 0.05 rad, so its amplitude is much lower than the amplitude of the changing of required output.

Using asymptotic output tracking control, a reference with a slope of 4 rad/s can be tracked without problems, but the overshoot in the output becomes higher with the given parameters if this slope is steeper.

4 Conclusion

The methods of nonlinear systems and control theory were applied to a simple musculoskeletal system with one joint and two muscles. Two types of controllers were designed: a linear pole-placement servo designed for the locally linearized model, and a nonlinear controller based on asymptotic output tracking method.

Our initial hypothesis was that the nonlinear asymptotic output tracking control gave a better performance than the pole-placement servo in the nonlinear model, especially when the range of the movement is wide. The simulation results showed us that this hypothesis is true: nonlinear control gives better performance. Its response is much faster than the pole-placement servo, it is able to track a continuously changing reference output, with minimal overshooting. The other advantage of the nonlinear control is that it is able to take into account the whole nonlinear behavior of musculoskeletal system and requires less input energy.

The overshoot of the nonlinear controller can be reduced with the proper choice of the c_i design parameters. When the value of c_0 is much higher than the value of c_1, the overshoot of asymptotic output tracking is smaller, but the output oscillates more.

The computation and simulation time of asymptotic output tracking controller was high due to the complexity of the nonlinear expressions in the feedback law (simulation of a 2.5 sec movement required 1 min on a Pentium III 600MHz, 128 MB RAM), while pole-placement controller simulation only required 5 sec.

The results suggest that linear control can be applied when the movement range is small or efficient computation is very important. But if the motion range is wide or the reference input is a more complex function of time, then the application of nonlinear control theory becomes necessary to provide satisfactory reference tracking.

Acknowledgements

G. Sz. is a grantee of the Bolyai János Research Scholarship of the Hungarian Academy of Sciences. The support of the Hungarian Scientific Research Fund through grants T042710, F046223, T034548 is gratefully acknowledged.

Appendix

Values of the applied model constants are shown in table 2. d_f is the moment arm of the flexor muscle. d_f^{prox} is the distance between the flexor muscle's origin and joint. d_f^{dist} is the distance between the flexor muscle's insertion and joint.

Table 2. Value of constants

const.	dim.	value	const.	dim.	value
τ_{act}^{f}	[s]	0.012	τ_{act}^{e}	[s]	0.012
β_f	[1]	0.5	β_e	[1]	0.5
c_f	[1]	-3.19	c_e	[1]	-3.19
$l_{CE,f}^{opt}$	[m]	0.3	$l_{CE,e}^{opt}$	[m]	0.3
$F_{PE,f}^{max}$	[1]	0.5	$F_{PE,e}^{max}$	[1]	0.5
$l_{T,f}^{slack}$	[m]	0.1	$l_{T,e}^{slack}$	[m]	0.1
$k_{T,f}$	[N/m]		$k_{T,e}$	[N/m]	
$s_{T,f}$	[Ns/m]	10000	$s_{T,e}$	[Ns/m]	10000
$z_{T,f}$	[Ns^2/m]	1	$z_{T,e}$	[Ns^2/m]	1
$F_{max,f}$	[N]	1000	$F_{max,e}$	[N]	1000
d_f	[m]	0.05	d_e	[m]	0.05
d_f^{prox}	[m]	0.2	d_e^{prox}	[m]	0.2
d_f^{dist}	[m]	0.05	d_e^{dist}	[m]	0.05
g	[m/s^2]	-9.81	l_{COM}	[m]	0.12
m	[kg]	2	Θ	[kgm^2]	0.015

References

[1] B.M. van Bolhuis and C.C.A.M. Gielen, A comparison of models explaining muscle activation patterns for isometric contractions, *Biological Cybernetics*, 81, 1999, 249–261.

[2] P.E. Cargo and J.T. Mortimer and P.H. Peckham, Closed-loop control of force during electrical stimulation of muscle, *IEEE Transactions on Biomedical Engineering*, 27(6), 1980, 306–311.

[3] P.E. Cargo and P.H. Peckham and G.B. Thrope, Modulation of muscle force by recruitment during intramuscular stimulation, *IEEE Transactions on Biomedical Engineering*, 27(12), 1980, 679–684.

[4] D. Csercsik and Cs. Fazekas and K.M. Hangos, Dynamical Analysis and Control of a Simple Nonlinear Limb Model, *Proc. of EMBEC 2005*, Prague, Czech Republic, 2005.

[5] V. De Sapio and J. Warren and O. Khatib and S. Delp, Simulating the task-level control of human motion: a methodology and framework for implementation, *Visual Computer*, 21, 2005, 289–302.

[6] K. M. Hangos and I. T. Cameron, *Process Modelling and Model Analysis*, (London: Academic Press, 2001).

[7] H. Hatze, The Complete Optimalization of Human Motion, *Mathematical Bioscience*, 28, 1976, 99–135.

[8] A.V. Hill, The Heat of Shortening and the Dynamic Constant of Muscle, *Proc. R. Soc. Lond. B*, 126, 1938, 136–195.

[9] A. Isidori, *Nonlinear Control Systems*, (Berlin: Springer, 1999).

[10] G. Khang and F.E. Zajac, Paraplegic Standing Controlled by Functional Neuromuscular Stimulation: Part I - Computer Model and Control-System Design, *IEEE Transaction on Biomedical Engineering*, 36, 1989, 873–884.

[11] H. van der Kooij and R. Jacobs and B. Koopman and H. Grootenboer, A multisensory integration model of human stance control, *Biological Cybernetics*, 80, 1999, 299–308.

[12] H. Nijmeijer and A. J. Van der Schaft, *Nonlinear Dynamical Control Systems*, (New York: Springer, 1990).

[13] M.G. Pandy and B.A. Garner and F.C. Anderson, Optimal control of non-balistic muscular movements: a constraint-based performance criterion for rising from chair, *Journal of Biomechanical Enginnering*, 117, 1995, 15–26.

[14] D.K. Peterson and H.J. Chizeck, Linear Quadratic Control of a Loaded Agonist-Antagonist Muscle Pair, *IEEE Transactions on Biomedical Engineering*, 34(10), 1987, 790–796.

[15] R. Riener and T. Fuhr, Patient-Driven Control of FES-Supported Standing Up: A Simulation Study, *IEEE Transactions on Rehabilitation Engineering*, 6(2), 1998, 113–124.

[16] Arjan van der Schaft, *L2-Gain and Passivity Techniques in Nonlinear Control*, (Berlin: Springer, 2000).

[17] A.J. van Soest and M.F. van Bobbert, The Contribution of Muscle Properties in the Control of Explosive Movements, *Biological Cybernetics*, 69, 1993, 195–204.

[18] A.J. Van Soest and P.A. Huijing and M. Solomonow, The effect of tendon on muscle force in dynamic isometric contractions: a simulation study, *Journal of Biomechanics*, 28, 1995, 801–807.

[19] D.G. Thelen and F.C. Anderson and S.L. Delp, General dynamic simulation of movement using computed muscle control, *Journal of Biomechanics*, 36, 2003, 321–328.

[20] F.E. Zajac, Muscle and tendon: properties, models, scaling and application to biomechanics and motor control, *CRC Critical Reviews in Biomedical Engineering*, 17, 1989, 359–411.

[21] V.M. Zatsiorsky, *Kinetics of Human Motion*, (Champaign, IL: Human Kinetics, 2002).