

modal amplitudes in the exact output by properly selecting the eigenvalues of the D matrix in (7). It is interesting at this point to note that (7) can be interpreted as a Luenberger type observer. Because of these special features, the lower order model is viewed as a *generalized aggregated model*.

A method of using this lower order generalized aggregated model for determining a suboptimal control law (that solves an output regulator problem) is explained.

Because the outputs of the exact and lower order generalized aggregated models are tolerably close under the application of any arbitrary inputs, it is reasonable to expect that the control law that optimally controls the output of the lower order model should be able to control the exact system also and that the resulting closed-loop system is asymptotically stable. However, it must be pointed out that stability of the closed-loop system is not guaranteed.

Expressions are given for the performance degradation resulting from the use of the suboptimal control policy.

A numerical example is given to illustrate the results developed in the paper. This example shows that it is possible to lessen the computational complexity associated with the higher order optimal control problem.

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Sampling in the Human Motor Control System

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Abstract—Some results of a physiological investigation of the sampling hypothesis in the human motor control system are presented in this paper. The hypothesis of a proprioceptively open loop system at the initiation of voluntary effort is not supported by data from ankle rotation. No discontinuity in the monosynaptic pathway (primary afferent fiber to alpha motoneuron) is observed during random isometric step tracking using the H reflex as test signal in the gastrocnemius-soleus reflex arc. This would indicate that for the ankle control system, the hypothesis of sampling at the alpha motoneuron, as proposed by Navas and Stark for wrist rotation, is not valid. The sampling behavior in the human motor system, if it exists, must be of the central origin.

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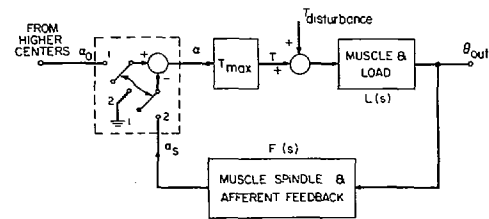


Fig. 1. Navas-Stark model for pursuit tracking of unpredictable inputs [4].

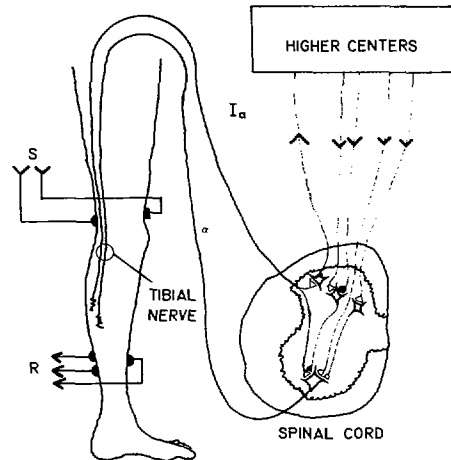


Fig. 2. Neural pathways for H reflex and supraspinal control. Stimulation is applied unifically by S to the posterior tibial nerve in the popliteal fossa, and the reflex pathway runs up to the spinal cord by I_α afferent fibers and out to the gastrocnemius-soleus muscle by α efferent motor fibers. Other fibers of the tibial nerve are not shown. Recording R is from the soleus muscle.

I. INTRODUCTION

The concept of the human operator performing tracking as a sampled data system has a long history [1]-[3]. It has been suggested that the response of the human operator in a control system is not continuous but intermittent. One recent model is that proposed by Navas and Stark [4] for pursuit tracking of unpredictable inputs with wrist rotation. Their model is reproduced in Fig. 1. The alpha motoneuron (AMN), enclosed in the dotted box, controls the contraction of the muscle. This force, summed at the joint with external forces and those of other muscles, reacts against external loading and the internal viscoelasticity of the muscle itself and causes rotation of the joint. The resulting change in muscle length is sensed by the muscle spindles and fed back to the AMN to form a negative-feedback position servo.

The Navas-Stark hypothesis is that the AMN, during pursuit tracking of an unpredictable target, does not act like a summing amplifier for sensory and central inputs but acts rather like a switching/sampling device. Some of their remarks [4] are quite pertinent to the present paper.

The behavior of the motor coordination system would occur as follows: Voluntary movements are preprogrammed in the higher centers and signaled as a whole. At the time of command release (position 1, Figure 1 of this paper) spindle afferent control is reduced and higher control of the alpha efferents is fully turned on. The system is operating open loop, characterized by high gain, low damping, and possible postural drift, during a short period of time, about 100 msec. Only apparent viscosity opposes motion; however, regulation comes from proprioceptive feedback and from the apparent viscosity. After the motion has been substantially completed, the spindle afferent control of the alpha efferent pathway is fully turned on and the gamma efferent may be altered for a new set point; this corresponds to position 2 in Figure 1.

In all the papers where sampled-data models of the human operator have been presented, direct physiological evidence of the sampling mechanism in the motor system has been lacking. We have reexamined this issue in the

hope of specifying the location of the sampler and the physiological mechanism involved.

II. METHOD OF SYSTEM IDENTIFICATION

To identify a system, one must be able to form some hypothesis, either *a priori* or *ad hoc*, as to its structure. In dealing with input-output experimental data, no unique relationship will exist and therefore the structure must be determined by other means.

Much experimentation has shown that the negative feedback arrangement of Fig. 1 has strong anatomical and physiological support. In fact it has been amply demonstrated [5], [6] that the large spindle-afferent fibers (I_a fibers) make monosynaptic connections with the alpha motoneuron of the muscle in which those spindles are located. This work was confirmed in humans [7], [8] and it was shown that this monosynaptic reflex could be elicited simply and precisely.

Fig. 2 shows the anatomical organization of the proprioceptive feedback loop involving the extensor muscle (gastrocnemius-soleus) of the foot. An electrical shock delivered to the popliteal fossa where the tibial nerve lies near the skin can elicit an afferent volley in the I_a sensory fibers without directly exciting the alpha motor fibers. This is the Hoffmann reflex or *H* reflex [7]-[11].

The stimulus excites a synchronized volley of action potentials in the I_a afferents that ascend to the spinal cord, synapse with the alpha motoneurons, and descend to excite the muscle. Approximately 30 ms after the stimulus, the volley may be recorded electrically as the electromyogram (EMG) in the muscle and is called the *H* wave. Quite plainly, the amplitude of the *H* wave will be a function of the stimulus intensity and, more importantly, of the gain of the I_a -alpha efferent pathway.

Thus, we see that the Navas-Stark model is experimentally testable using the *H* reflex. If there is indeed a sampler at the alpha motoneuron, then measuring the reflex gain should reveal it. When the motoneuron switch lies in position 1 (Fig. 1), then the feedback pathway is open and the reflex gain should be much lower (close to zero) than when the switch is in position 2, and the feedback pathway is effective. The Navas-Stark hypothesis is that the sampler is synchronized with the movement, at the movement's initiation being in position 1 and at the end in position 2. Thus, a vigorous contraction of the gastrocnemius-soleus muscle should show the *H* reflex blocked initially and subsequently restored to its resting level.

III. EXPERIMENTAL PROCEDURE

The experiments were performed on nine human subjects ranging in ages from 21 to 36 years. A subject was seated in a chair with his right leg extended and the foot strapped to a fixed plate for measuring torque. A visual display of the level of foot torque was provided on one channel of a dual-beam scope. The position of a target (second spot on the scope) was controlled by an IBM 1800 computer, which ran the experiments and recorded the data. The computer sampled and recorded the torque and the gastrocnemius-soleus muscle EMG. Electrical stimuli were applied to the subject's tibial nerve by means of cutaneous electrodes located posteriorly in the popliteal fossa and anteriorly above the knee.

The basic idea of the experiment was to test the excitability of the I_a -alpha motoneuron synapse over the full time course of a repeated voluntary effort by means of an *H* reflex elicited at different times during such efforts. The subject was instructed to track a target that jumped back and forth between two positions at random 7-10-s intervals. The subject tracked the target with his own spot by changing the amount of force he exerted on the foot plate. After the target jumped to a new position, the stimulator was triggered by the computer. By varying the delay between the target jump and the stimulus, and allowing for the natural variability of the subject's response time, the *H* reflex could be elicited over the entire interval of the voluntary movement. (For a more complete discussion of the experimental procedure, see [11], [12].)

IV. RESULTS

At submaximal stimulus inputs, the peak-to-peak amplitude of the *H* wave is a function of the static muscle activity, i.e., the static or initial foot torque. The *H*-wave amplitude, recorded when the subject is tracking

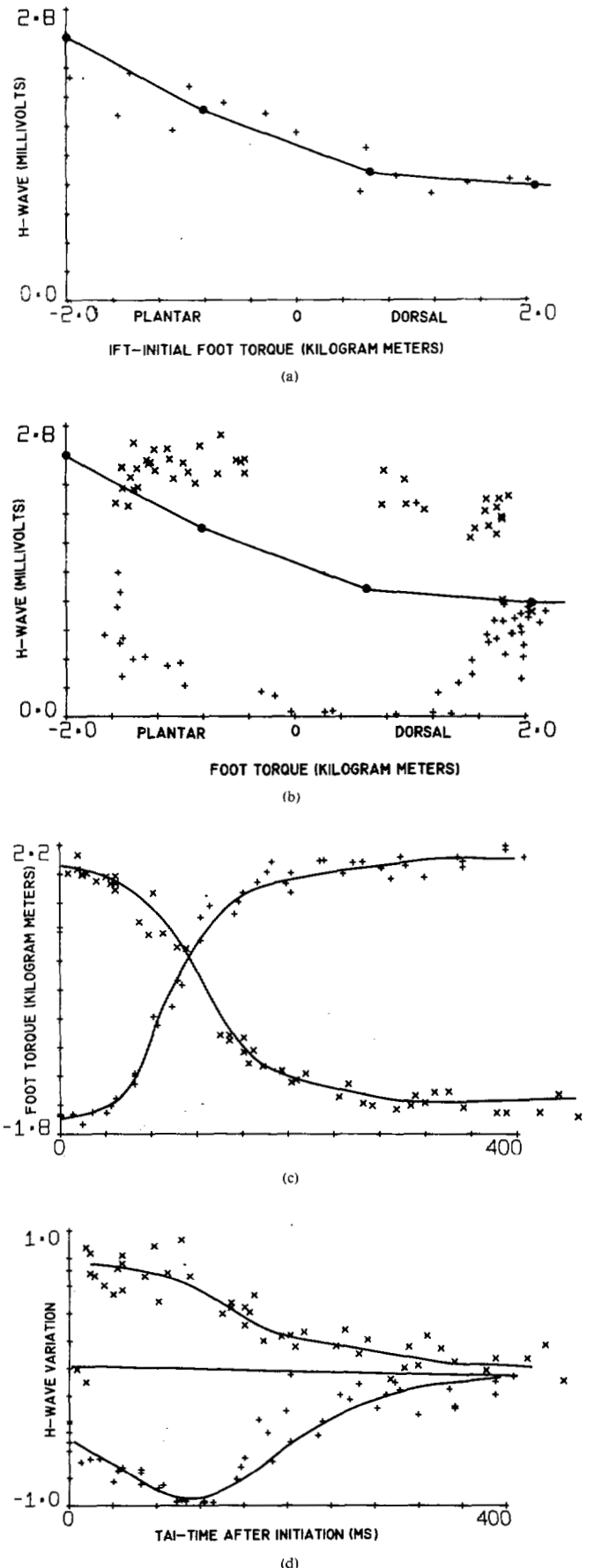


Fig. 3. Response of subject GCA in unpredictable (time of occurrence), isometric effort step tracking (+ dorsiflexion, x plantarflexion step effort).

a step input by changing his muscle activities, must be corrected for this static influence on H wave to determine dynamic facilitation or inhibition of the motoneuron pool. Fig. 3 shows the results of one typical experiment. The four parts of this figure are as follows.

Fig. 3(a): This figure shows the dependence of the amplitude of the H wave on the level of initial foot torque. This dependence is almost always monotonic, increasing in amplitude with plantarflexion and decreasing with dorsiflexion. The solid line in the figure is a piecewise linear normalizing curve that is fitted to the data by eye.

Fig. 3(b): This figure shows the amplitude of the H wave measured during step tracking. In this and the succeeding parts of these figures, the symbol $+$ will denote a measurement made during a dorsiflexional step effort and the symbol \times will denote a measurement made during a plantarflexional step effort. The solid line shows the normalizing curve from Fig. 3(a). Notice that the dynamic plantarflexion points lie on or above the normalizing curve while dynamic dorsiflexion points lie on or below it.

Fig. 3(c): The time at which the H reflex is recorded is under computer control but the time of the initiation of the step effort varies with each data record. Therefore, a variable time after initiation (TAI) is defined, which is the interval of time between the initiation of the effort and the recording of the H wave. Fig. 3(c) shows the value of the foot torque in step tracking at the times when the H wave was recorded versus time after initiation of the movement.

Fig. 3(d): This shows the H -wave variation ΔH , during tracking, plotted against TAI. The positive and negative values of the ratio ΔH denote facilitation and inhibition, respectively, with respect to the static value of the H wave for the same foot torque. The equation for ΔH is

$$\Delta H = (H_m - H_n)/H_n$$

where H_m is the measured value of the H wave during dynamic tracking and H_n is the value of normalizing curve [Fig. 3(a)] at the same foot-torque level.

Fig. 4 shows two responses recorded directly from an oscilloscope display. Marked facilitation during plantarflexion and inhibition during dorsiflexion is clear.

V. DISCUSSION

Before discussing the results of our experiments, some differences between the present experimental procedure and the Navas-Stark work should be made clear. Whereas Navas and Stark studied wrist rotation that involves pronator and supinator muscles, we have investigated movements around the ankle joint involving gastrocnemius-soleus and anterior tibial muscles. The physiological roles for these systems are different and so are their mechanical characteristics. However, the reflex mechanisms are presumably the same. Most of the Navas-Stark work was done with no external loading on the movement, while our experiments were done isometrically.

Fig. 3 shows that the monosynaptic reflex in the gastrocnemius-soleus motoneuron pool is facilitated during a dynamic plantar effort. The facilitation precedes the initiation of the effort and decreases as the effort is completed. During the dorsal dynamic effort, the gastrocnemius-soleus motoneuron pool is inhibited throughout, and maximum inhibition is at the time when the anterior tibial muscle is most active.

These experiments indicate that the monosynaptic pathway (I_a to AMN) of the gastrocnemius-soleus motoneuron pool is continuously excitable during a plantarflexion effort. This contradicts the Navas-Stark hypothesis that at the initiation of the voluntary movement, the system is operating proprioceptively open loop. Since no discontinuity in reflex excitability is observed during the course of the plantar movement, this would indicate that sampling does not occur at the AMN or any other peripheral point. Consequently, the so-called sampling behavior in the human motor system, if it exists, must be of central origin. However, at this stage, it is not clear whether the sampling hypothesis is necessary to explain the limited amount of data (peak in the gain characteristics in complex sine-wave tracking, pulse-like response in slow ramp tracking, and short-width unpredictable step response) that supports sampling behavior. Some nonlinearity in the system may give rise to these characteristics. Much more has to be learned about the human motor system before this question can be settled.

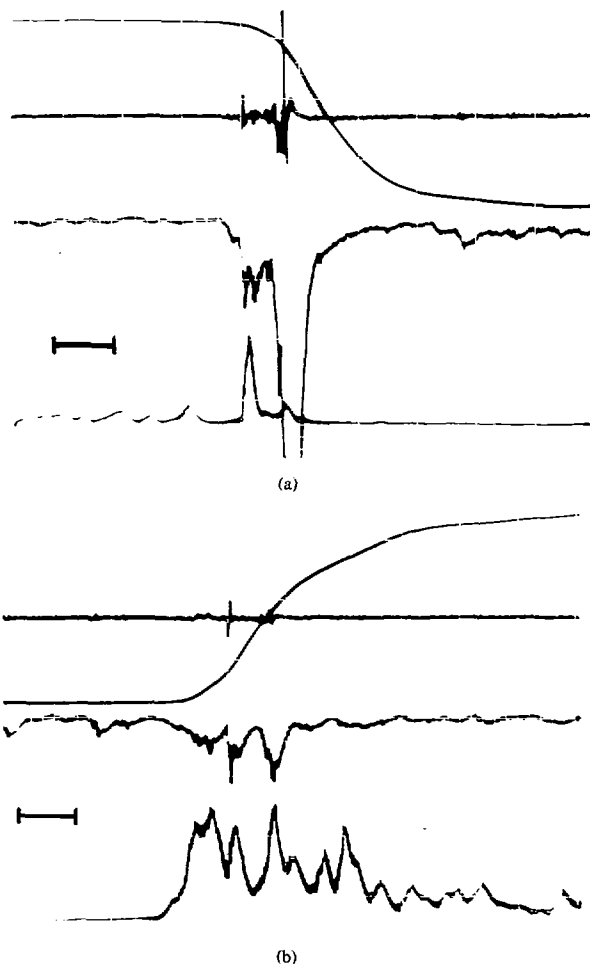


Fig. 4. Typical individual responses in isometric step tracking. The four traces from top to bottom are foot torque (1.2 kg · m/unit); H reflex (0.8 mV/unit); filtered EMG of the gastrocnemius-soleus muscle (GSM); and filtered EMG of the anterior tibial muscle (ATM) (2.0 V/unit). The downward foot torque deflection is in the plantar direction. Strong facilitation during plantar effort and nearly total inhibition during dorsal effort are clearly shown. The first marker in the H reflex trace is stimulus artifact, which is also reflected in both filtered EMGs. Time scale is 50 ms/unit.

VI. CONCLUSIONS

In this paper, it has been shown that the monosynaptic pathway from the spindle primary afferent fibers to the alpha motoneuron pool of the gastrocnemius-soleus muscle is continuously excitable during a plantarflexion effort in unpredictable step effort tracking. This is contrary to the hypothesis of a proprioceptively open loop system at the initiation of a voluntary effort and also this evidence is contrary to the sampling hypothesis at the alpha motoneuron as proposed by Navas and Stark. However, these results are not to be taken as evidence against all sampled data models that have been proposed for the human motor system. The data merely suggest that sampling in the human motor system, if present, is not at any peripheral location but must be of central origin. At this stage, it is not clear whether the sampling hypothesis is essential to explain the limited amount of data available.

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Design of Time-Variable Multivariable Systems by Decoupling and by the Inverse

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Abstract—System decoupling by state variable feedback is demonstrated for the time-variable case. Conditions under which decoupling is possible are formulated and the matrices effecting decoupling are given. The inverse of a general time-variable multivariable system is also determined. By applying this inverse a new synthesis method is developed.

INTRODUCTION

Decoupling by state-variable feedback has been considered by several authors for systems with constant coefficients [1]–[3]. Using the definitions of Falb–Wolovich [3] and extending them in an appropriate way, the problem of decoupling is solved here for systems with time-variable coefficients.

The determination of the inverse system has also received some attention. The inverse was first introduced by Brockett [4] for single-input single-output systems with constant coefficients. The concept of the inverse system has subsequently been generalized along two different lines; the extension to multivariable systems (with constant coefficients) was achieved in [3] and the inverse of time-variable single-input single-output systems was given in [5], [6]. In the present paper the inverse for the most general case, i.e., for multivariable and time-variable systems is determined.

The application of the inverse leads to a new synthesis method for time-variable multivariable systems using linear transfer operators in connection with elementary operations (like addition, multiplication, and inversion).

DEFINITIONS AND NOTATIONS

The class of time-variable systems to be considered is described by the pair of vector equations

$$\begin{aligned} \dot{x} &= A(t)x + B(t)u \\ y &= C(t)x \end{aligned} \tag{1}$$

where the input vector u and the output vector y have equal dimension m ; x is the state vector of dimension n ($n \geq m$). The matrices $A(t)$, $B(t)$, and $C(t)$ are order compatible with the vectors u , y , and x .

The original input u is supposed to be generated by state variable feedback plus a feedforward term:

$$u = F(t)x + G(t)v \tag{2}$$

where v is a new m input vector and $F(t)$ is an $m \times n$ matrix and $G(t)$ a nonsingular $m \times m$ matrix.

Substitution of (2) into (1) yields

$$\begin{aligned} \dot{x} &= [A(t) + B(t)F(t)]x + B(t)G(t)v \\ y &= C(t)x. \end{aligned} \tag{3}$$

It is well known that for time-variable systems the following operator is useful. Expressed in terms of system (1) this operator has the form

$$L_A^k C_i = (L_A^{k-1} C_i)' + (L_A^{k-1} C_i)A, \quad k = 1, 2, \dots, n \tag{4}$$

with

$$L_A^0 C_i = C_i$$

The dot denotes a differentiation with respect to time; C_i is the i th row of C . It can be shown that the following relation is true between the operator (4) applied to system (1) and to system (3), respectively:

$$L_{A+BF}^k C_i = L_A^k C_i + \sum_{j=0}^{k-1} L_{A+BF}^{k-1-j} [(L_A^j C_i)BF], \quad k = 0, 1, \dots, n. \tag{5}$$

(By definition a series shall be zero if the lower index is greater than the upper one, also in (7) and (9).)

In extension of [3] the following definition for the "differential order" ρ_i of a time-variable multivariable system is proposed.

Definition 1: Let for all t

$$(L_A^{\gamma_i} C_i)B = 0, \quad 0 \leq \gamma_i \leq \rho_i - 2$$

and

$$(L_A^{\rho_i-1} C_i)B \neq 0$$

then the "differential order" is ρ_i , $i = 1, 2, \dots, m$. Application of definition 1 to (5) results in

$$L_{A+BF}^k C_i = L_A^k C_i, \quad k = 0, 1, \dots, \rho_i - 1$$

and

$$L_{A+BF}^k C_i = L_A^k C_i + \sum_{j=\rho_i-1}^{k-1} L_{A+BF}^{k-1-j} [(L_A^j C_i)BF], \quad k = \rho_i, \rho_i + 1, \dots, n.$$

DECOUPLING

Repeated differentiation of the i th output variable y_i ($i = 1, 2, \dots, m$) [7] and application of (6) yields

$$y_i^{(k)} = [L_{A+BF}^k C_i]x, \quad k = 0, 1, \dots, \rho_i - 1$$

and

$$\begin{aligned} y_i^{(k)} &= [L_{A+BF}^k C_i]x + \sum_{p=\rho_i}^k \binom{k-p}{0} [(L_{A+BF}^{p-1} C_i)BG]^{(k-p)}v + \dots \\ &+ \sum_{p=\rho_i}^{k-j} \binom{k-p}{j} [(L_{A+BF}^{p-1} C_i)BG]^{(k-p-j)}v^{(j)} + \dots \\ &\dots + \binom{k-\rho_i}{k-\rho_i} (L_{A+BF}^{\rho_i-1} C_i)BGv^{(k-\rho_i)}. \end{aligned} \tag{7}$$

Denoting the coefficients of the characteristic time-variable equation with $\alpha_{i,k}$ the scalar equation for y_i [7] can be obtained by eliminating x in (7):

$$y_i^{(n)} - \alpha_{i,n}y_i^{(n-1)} - \dots - \alpha_{i,1}y_i = M_i(v) \tag{8}$$

where

$$\begin{aligned} M_i(v) &= - \left[\sum_{l=0}^{n-\rho_i} \alpha_{i,n-1+l} \sum_{p=\rho_i}^{n-1} \binom{n-p-1}{0} [(L_{A+BF}^{p-1} C_i)BG]^{(n-p-1)} \right] v - \dots \\ &- \left[\sum_{l=0}^{n-\rho_i-j} \alpha_{i,n-1+l} \sum_{p=\rho_i}^{n-1-j} \binom{n-p-1}{j} [(L_{A+BF}^{p-1} C_i)BG]^{(n-p-1-j)} v^{(j)} \right] \\ &- \dots - \alpha_{i,n+1} \binom{n-\rho_i}{n-\rho_i} (L_{A+BF}^{\rho_i-1} C_i)BGv^{(n-\rho_i)}, \quad \alpha_{i,n+1} = -1. \end{aligned} \tag{9}$$

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