Solving the equations of motion with a numerical method

The equations of motion have relatively simple **analytical solutions** for motion with constant velocity or with constant acceleration. In the first case, the solution is a linear function for the position and in the second case a quadratic function.

The equations of motion can also be solved using a **numerical approach**. The numerical approach is particular useful when the acceleration is not constant.

The basic idea of the numerical solution is to compute the next value of a quantity that is changing with a known rate with an **iterative equation** of the type:

$$\text{new value of the quantity} = \text{previous value of the quantity} + \text{change of the quantity}$$

$$+ \text{rate of change of the quantity} \times \text{time increment}$$

$$a_{y,t} = \frac{\sum F_{y,t}}{m}$$

Knowing the sum of the forces at a certain instant *t*, it is possible to compute the acceleration at that instant using **Newton's fundamental law of motion**...

$$\frac{dv_y}{dt} = a_{y,t}$$

By definition, the **instantaneous rate of change of velocity** is **acceleration**...

$$\frac{\Delta v_y}{\Delta t} \approx a_{y,t}$$

An **approximation**, for a finite time interval Δt (the rate of change of velocity is acceleration...)

$$\frac{v_{y,t+\Delta t} - v_{y,t}}{\Delta t} = a_{y,t}$$

Expressing the **change in velocity** from instant t to instant $t + \Delta t$...

$$v_{y,t+\Delta t} - v_{y,t} = a_{y,t} \times \Delta t$$

$$v_{y,t+\Delta t} = v_{y,t} + a_{y,t} \times \Delta t$$

Expressing the **new value of velocity** on instant $t + \Delta t$ as the previous value on instant *t* plus the change in velocity, measured as the product of acceleration on the previous instant multiplied by the time increment...

$$\frac{dy}{dt} = v_y$$

By definition, the **instantaneous rate of change of position** is **velocity**...

$$\frac{\Delta y}{\Delta t} \approx v_y$$

An **approximation**, for a finite time interval Δt (the rate of change of position is velocity...)

$$\frac{y_{t+\Delta t} - y_t}{\Delta t} = v_y$$

Expressing the **change in position from instant *t* to instant $t + \Delta t$...**

$$y_{t+\Delta t} - y_t = v_y \times \Delta t$$

$$y_{t+\Delta t} = y_t + v_y \times \Delta t$$

Expressing the **new value of position** on instant $t + \Delta t$ as the previous value on instant *t* plus the change in position, measured as the product of velocity on the previous instant multiplied by the time increment...

Synthesis of the numerical method to solve the equations of motion:

$$a_{y,t} = \frac{\sum F_{y,t}}{m}$$

$$v_{y,t+\Delta t} = v_{y,t} + a_{y,t} \times \Delta t$$

$$y_{t+\Delta t} = y_t + v_{y,t} \times \Delta t$$

This method is known as **Euler method**...

A better numerical method to solve the equations of motion (particularly for oscillating forces!):

$$a_{y,t} = \frac{\sum F_{y,t}}{m}$$

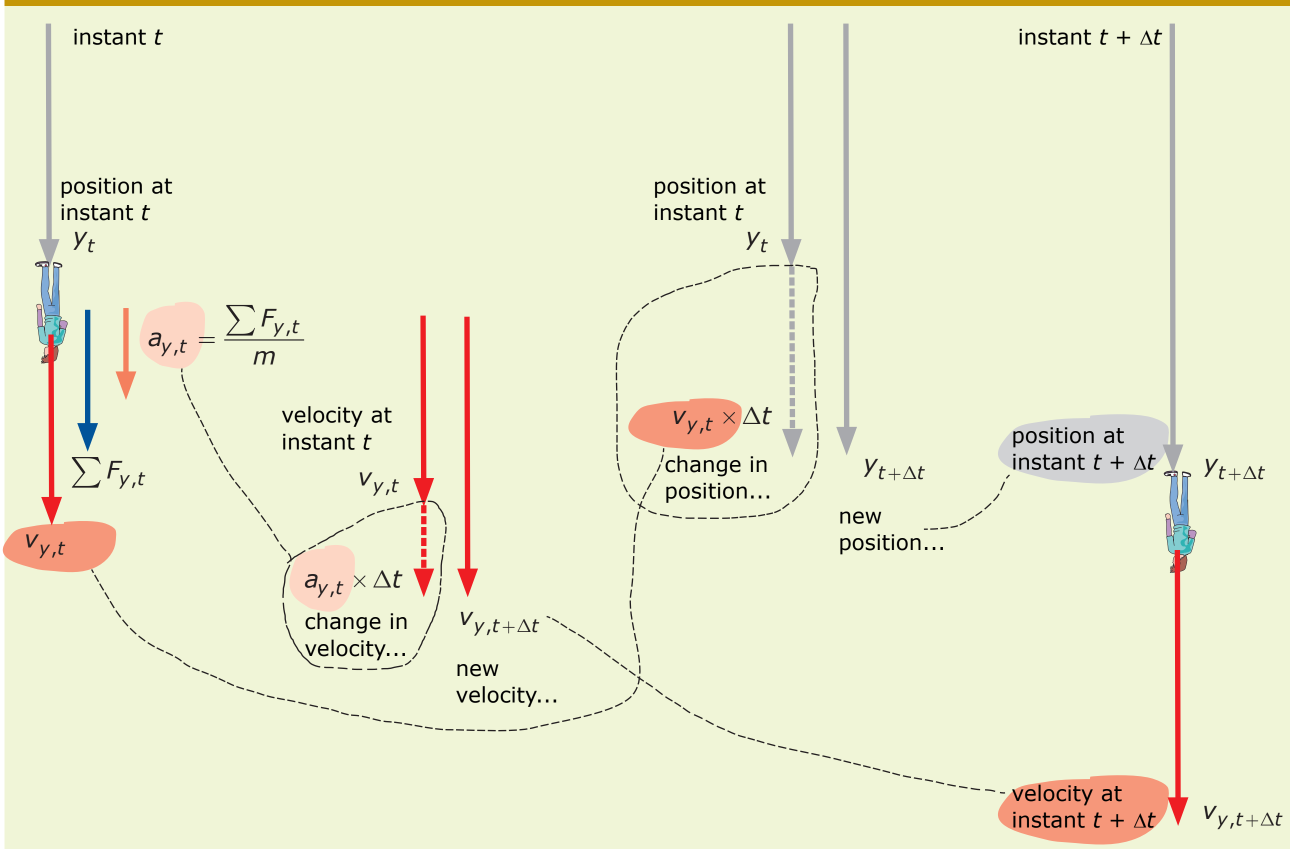
$$v_{y,t+\Delta t} = v_{y,t} + a_{y,t} \times \Delta t$$

$$y_{t+\Delta t} = y_t + v_{y,t+\Delta t} \times \Delta t$$

To **solve** the equations **numerically**, it is necessary to **know how to compute the force**, the mass, and the **initial values** of the position and of the velocity.

This method is known as **Euler-Cromer method**...

Euler method (numerical solution of the equations of motion)



Euler-Cromer method (numerical solution of the equations of motion)