Chapter 33
Improving Learning in Science and Mathematics with Exploratory and Interactive Computational Modelling

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Abstract Scientific research involves mathematical modelling in the context of an interactive balance between theory, experiment and computation. However, computational methods and tools are still far from being appropriately integrated in the high school and university curricula in science and mathematics. In this chapter, we present a new way to develop computational modelling learning activities in science and mathematics which may be fruitfully adopted by high school and university curricula. These activities may also be a valuable instrument for the professional development of teachers. Focusing on mathematical modelling in the context of physics, we describe a selection of exploratory and interactive computational modelling activities in introductory mechanics and discuss their impact on student learning of key physical and mathematical concepts in mechanics.

1 Introduction

Science is an evolving structure of knowledge based on hypotheses and models which lead to theories whose explanations and predictions about the universe must be consistent with the results of systematic and reliable experiments (see, e.g. Chalmers 1999; Feynman 1967). The process of creating scientific knowledge is an interactive blend of individual and group reflections which involve modelling

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processes that balance theory, experiment and computation (Blum et al. 2007; Schwartz 2007; Slooten et al. 2006). This cognitive frame of action has a strong mathematical character, since scientific reasoning embeds mathematical reasoning as scientific concepts and laws are represented by mathematical entities and relations. In this process, computational modelling plays a key role in the expansion of the science and mathematics cognitive horizon through enhanced calculation, exploration and visualisation capabilities.

Although clearly linked to real world phenomena, science and mathematics are thus based on abstract and subtle conceptual and methodological frameworks which change along far from straightforward evolution timelines. These cognitive features make science and mathematics difficult subjects to learn, to develop and to teach. In an approach to science and mathematics education meant to be effective and in phase with the rapid scientific and technological development, an early integration of computational modelling in learning environments which reflect the exploratory and interactive nature of modern scientific research is of crucial importance (Ogborn 1994). However, computational knowledge and technologies, as well as exploratory and interactive learning environments, are still far from being appropriately integrated into high school and university curricula in science and mathematics. As a consequence, these curricula are generally outdated and most tend to transmit to students a sense of detachment from the real world. These are contributing factors to the development of negative views about science and mathematics education, leading to an increase in student failure.

Physics is a good illustrative example. Consider the general physics courses taken by first year university students. These are courses which usually cover a large number of difficult physics topics following a traditional lecture plus laboratory instruction approach. Due to a lack of understanding of fundamental concepts in physics and mathematics, the number of students that fail in examination tests is usually very high. Moreover, many students that eventually succeed also reveal several weaknesses in their understanding of elementary physics and mathematics (Halloun and Hestenes 1985; Hestenes 1987; Hestenes et al. 1992; McDermott 1991; McDermott and Redish 1999).

Although it is clear that there are many reasons behind this problem, it is also clear the solution has to involve changes in the physics education model. Indeed, many research studies have shown that the process of learning can be effectively enhanced when students are involved in the learning activities as scientists are involved in research (Beichner et al. 1999; Handelsman et al. 2005; Keiner and Burns 2010; Mazur 1997; McDermott 1997; McDermott and Redish 1999; Redish 2004). In addition, several attempts have been made to introduce computational modelling in research-inspired learning environments. The starting emphasis was on professional programming languages such as Fortran (Bork 1967) and Pascal (Redish and Wilson 1993). Although more recently this approach has evolved to Python (Chabay and Sherwood 2008), it still requires students to develop a working knowledge of programming, a generally time-consuming and dispersive task which can hinder the process of learning physics. The same happens when using scientific computation software such as Mathematica and Matlab. To avoid overloading students with programming notions or syntax, and focus the learning process on the relevant physics and mathematics, several computer modelling systems were created, for example, Dynamical Modelling System (Ogborn 1985), Stella (High Performance Systems 1997), Easy Java Simulations (Christian and Esquembre 2007) and Modellus (Teodoro 2002).

In this chapter, we discuss how Modellus (see http://modellus.fct.unl.pt) can be used to develop exploratory and interactive computational modelling activities which can be adopted by high school and university curricula in science and mathematics as well as be a valuable instrument for the professional development of teachers. Focusing on mathematical modelling in the context of physics, we describe activities in introductory mechanics which were implemented in a new course component of the general physics course taken by first year biomedical engineering students at the Faculty of Sciences and Technology of the New Lisbon University (FCT/UNL). For mathematics education, these activities are relevant as concrete applications of mathematical modelling (Carson 1999; Garcia et al. 2006; National Research Council 1989).

2 Course Organisation, Methodology and Student Evaluation Procedures

Let us start by describing the implementation context for the computational modelling activities. The organisation, methodology and evaluation strategies used in general physics can serve as a model to be adapted to other areas of science and to mathematics.

The 2009 general physics course for biomedical engineering involved 115 students, 59 of them taking the course for the first time. The structure and programme themes were those of the 2008 edition (Neves et al. 2009). In the computational modelling classes, students were organised in groups of two or three, one group for each available computer. In each class, the groups worked on a set of five computational modelling activities conceived to be interactive and exploratory learning experiences about challenging but easily observed physical phenomena. An example is the motion of a swimmer in a river with a current (Neves et al. 2009). The teams were motivated to solve the problems on their own using the physical, mathematical and computational modelling guidelines provided by the class documentation. To ensure adequate working rhythm with appropriate conceptual, analytical and computational understanding, the students were continuously helped during the exploration of the activities.

All activities were created as computational modelling experiments with Modellus. Each class activity was presented in a PDF document, with text and embedded video support to help students both in class or at home in a collaborative online context based on the Moodle online learning platform. To design the activities, emphasis was placed on cognitive conflicts in the understanding of physical concepts, the manipulation of multiple representations of mathematical models and the interplay between the analytical and numerical approaches applied to solve problems in physics and mathematics. In this course, the majority of the supporting text and videos presented complete step-by-step instructions to build the Modellus mathematical models, animations, graphs and tables. After constructing the models, students explored the multiple representations available to answer several questions about the proposed general physics problems. Some activities involved modelling problems where students saw only videos of the Modellus animations or graphs. After this they constructed the mathematical models to reproduce the animations or graphs, and answer proposed questions. Modellus was particularly effective in these classes because of the following main advantages: (1) an easy and intuitive creation of mathematical models using standard mathematical notation, (2) the possibility to create animations with interactive objects that have mathematical properties expressed in the model and (3) the simultaneous exploration of images, tables, graphs and object animations.

The student evaluation procedures in the computational modelling classes involved group evaluation and individual evaluation. For each class, all groups had to build five Modellus models and complete a Moodle online test answering the questions of the corresponding activity PDF document. The individual evaluation consisted of the solution of two homework activities and a final test, both with new problems based on those covered in class but with only partial text and video instructions on how to build the models and solve the problems. Students also took pre-instruction and post-instruction Force Concept Inventory (FCI) tests (Hestenes et al. 1992) which did not count for their final classification. At the end of the semester, students answered a Likert scale questionnaire to access their degree of receptivity to this new computational modelling component of the general physics course.

3 Computational Modelling Activities with Modellus

Let us now discuss, as illustrative examples, two of the computational modelling activities about circular motion and oscillations, the theme opening the second part of the course. Again, these are thought not only from the point of physics but also from the point of view of mathematics in order to *help students make connections between different subjects*.

A particle in circular motion (representing, for instance, a runner going around a circular track) describes a circle of radius R, a mathematical curve defined by $x^2+y^2=R^2$ in a Cartesian reference frame Oxy whose origin is at the centre of the circle. In this frame, x and y are the Cartesian coordinates of the position vector $\vec{r}$. This vector has magnitude R and specifies where the particle is on the curve. As the particle moves around the circle, the magnitude R is kept constant but the direction of $\vec{r}$ changes with time. This direction is given by the angle θ that $\vec{r}$ makes with the Ox axis. The variables R and θ define the polar coordinates of $\vec{r}$. The coordinates x and y are also time dependent and are related to R and θ by trigonometric functions: $x = R\cos(\theta)$ and $y = \sin(\theta)$.

To explore circular motion, students started with uniform circular motion. When the circular motion is uniform, the particle traces one circle in every constant time interval T. This time interval is the period of the motion and its inverse $f=1/T$ is the frequency of the motion. The angle θ is then a linear parametric function of the time t, $\theta=\omega\, t+\theta_0$ where $\omega=2\pi/T$ is the motion angular frequency, measured in radians per second, and θ_0 is the initial direction of $\vec{r}$. The velocity $\vec{v}$ is tangent to the circular trajectory, always orthogonal to $\vec{r}$, and has constant magnitude $v=\omega\, R$. The acceleration $\vec{\alpha}$ has magnitude $\alpha=\omega^2\, R$ and a centripetal direction, that is, opposite to $\vec{r}$. The uniform circular motion is the composition of two simple harmonic oscillations: one along the Ox axis and the other along the Oy axis. These oscillations are characterised by the same amplitude $A=R$ and the same frequency $f=1/T$. The initial phase of the Ox oscillation is θ_0, and between them, there is a time-independent $\pi/2$ phase difference.

To model this type of motion, students had to recall what they learnt in the first part of the course during the computational modelling activities about vectors, parametric equations of motion, velocity and acceleration (Neves et al. 2009). Building on this prior knowledge, students were able to construct a model associating the Cartesian coordinates of $\vec{r}$ to the corresponding polar trigonometric functions with the angle θ given by the linear parametric equation $\theta=\omega\, t+\theta_0$. They were also able to define the coordinates of $\vec{v}$ and $\vec{\alpha}$ (see Fig. 33.1). This mathematical model was complemented with graphs and tables of the different coordinate variables as functions of time, and by an animation allowing direct manipulation of the independent parameters of the model, R, T, θ_0, as well as real time visual display of the trajectory of the moving particle, $\vec{r}$, $\vec{v}$, and $\vec{\alpha}$. The harmonic oscillatory motions along the coordinate axis were also represented (see Fig. 33.1). With this model, students were able to explore, visualise and reify the initially abstract physical and mathematical concepts associated with uniform circular motion. For example, by combining the information from the several different simultaneous representations, they analysed the motion of a particle tracing a circle of radius $R=150$ m every 2.5 min, and were able to compare $\vec{v}$ and $\vec{\alpha}$ as functions of time and to calculate these vectors at time $t=7$ min.

During these activities, students showed difficulties in distinguishing between a vector, like $\vec{v}$ or $\vec{\alpha}$, and its magnitude. They were also puzzled when asked to solve the same problem considering the angles measured in degrees instead of radians. Indeed, at first, students were frequently unable to create $\vec{v}$ and $\vec{\alpha}$ with the correct magnitude and direction. Similarly, they did not place the angle conversion factor in the correct place everywhere in the mathematical model. For example, in their corrrect attempt, they incorrectly multiplied the speed by $180/\pi$. To be able to correct the models and at the same time visualise the effect of the change in the animation and other model representations was for the students an essential advantage of the modelling process with Modellus in helping them to solve these learning difficulties.

Using and extending this trigonometric model, students were then able to construct a model in Modellus to estimate the solution to the following astronomical problem: What is the time interval between two successive oppositions of the Earth and Mars? To help students, we suggested the assumption of considering the motions

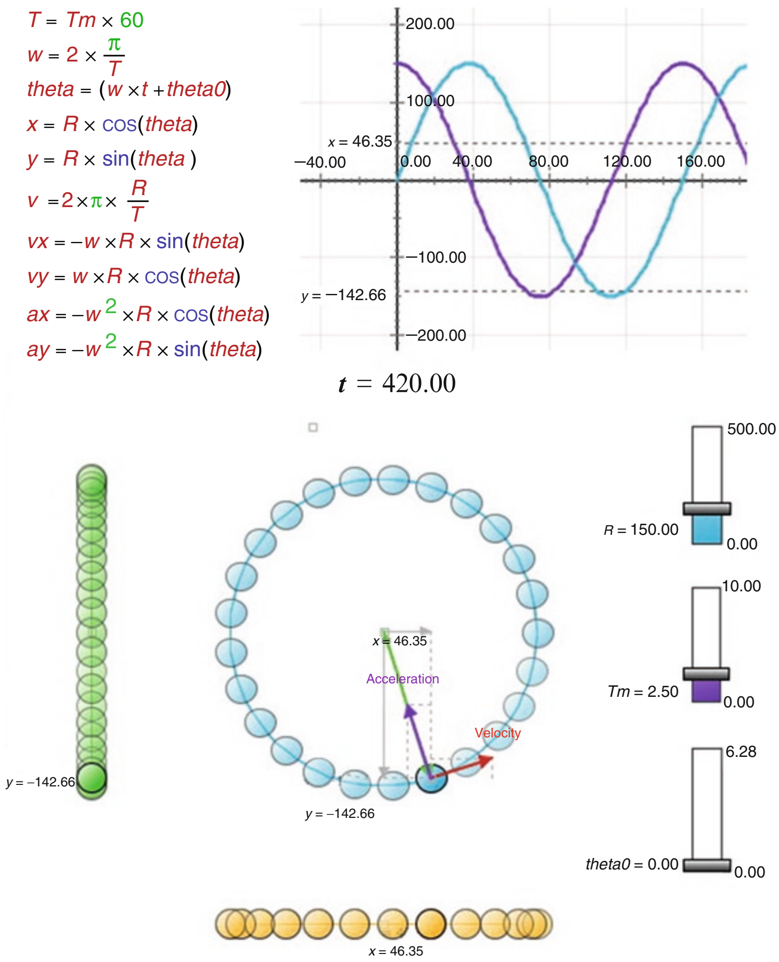

Fig. 33.1 Uniform circular motion: equations as seen in the Modellus Mathematical Model window, examples of coordinate-time graphs and the Modellus animation

of the Earth and Mars around the Sun to be uniform circular motions. We also taught them to use the average Earth–Sun distance (known as the astronomical unit and denoted by AU) as the distance scale for the problem. In this scale, the average Earth–Sun distance is simply 1 AU and the average Mars–Sun distance is 1.53 AU. Taking into account that the approximate motion periods of the Earth and Mars are, respectively, 1 year and 1.89 years and using the year as the unit of time, students were able to develop a mathematical model and an animation representing the motions of the Earth and Mars around the Sun. In the process, they were able to determine the angular velocities of both planets and the time interval between two successive oppositions. Using the conversion factors 1 AU = 1.50×10^8 km and

1 year = 3.15×10^7 s, they were also able to find in km/s the orbital velocities of the Earth and Mars at the time of the model first occurring opposition. To achieve the precision required by the Moodle online test, students used a position vector or velocity coincidence method. The adjustment of the numerical step was an important numerical technique students learned to apply to obtain animations with realistic trajectories and correct answers to the questions of this astronomical challenge.

4 Conclusions

In this chapter, we have shown how Modellus can be used to develop exploratory and interactive computational modelling activities for science and mathematics education. We have described examples in introductory mechanics which were implemented in the general physics course taken by first year biomedical engineering students at FCT/UNL. We have shown that during class, the computational modelling activities with Modellus were successful in identifying and resolving several student difficulties in key physical and mathematical concepts of the course. Of crucial importance in this process, was the possibility to have a real time visible correspondence between the animations with interactive objects and the object's mathematical properties defined in the model, and also the possibility of manipulating simultaneously several different representations such as graphs and tables. Thus with Modellus, students can be exploring authors of models and animations, and not just simple browsers of computer simulations.

The successful class implementation of the computational modelling activities was reflected in the student answers to a Likert scale questionnaire (see Fig. 33.2), with results improving slightly on those of the 2008 edition (Neves et al. 2008). Globally, students reacted positively to the activities, considering them to be helpful in the learning process of mathematical and physical models. For them, Modellus was easy enough to learn and user-friendly. In this course, students showed a clear preference to work in teams in an interactive and exploratory learning environment. The computational modelling activities with Modellus presented in PDF documents with embedded video guidance were also considered to be interesting and well designed. A natural sense of caution in relation to novelty and to evaluation procedures was nevertheless detected. Students also felt that the content load was heavy and that the available time spent on the computational modelling activities was insufficient.

In spite of global success during the class implementation phase, the FCI test results led to an average FCI gain of 22%, an indication that the general physics course with the computational modelling component is just performing as a traditional instruction course (Hake 1998). Although this performance score refers to the general physics course as a whole, the results of the questionnaire and students' opinions about the computational modelling component also indicate that some aspects of the implementation approach should be changed. In this context, possible ways forward are: (1) Increase the relative importance and value of the computational modelling component. (2) Reduce the heavy content load (as perceived by

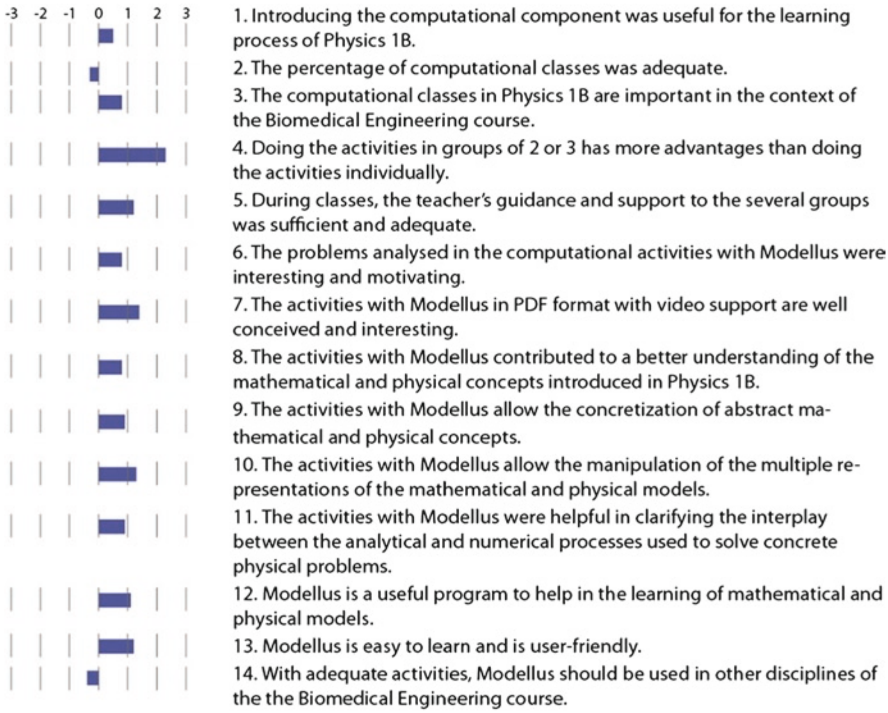

Fig. 33.2 Questionnaire and results, shown graphically by the average bar over all answers

students). (3) Increase time spent on the modelling tasks. (4) Choose problems more closely related with the specific subject of the student's course major. (5) Introduce less guided, more discovery-oriented instruction guidelines as well as computational modelling problem finding.

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References

Beichner, R., Bernold, L., Burniston, E., Dail, P., Felder, R., Gastineau, J., Gjertsen, M., & Risley, J. (1999). Case study of the physics component of an integrated curriculum. *Physics Education Research, American Journal of Physics Supplement, 67*, 16–24.

Blum, W., Galbraith, P., Henn, H.-W., & Niss, M. (Eds.). (2007). *Modelling and applications in mathematics education*. New York: Springer.

Bork, A. (1967). *Fortran for physics*. Reading: Addison-Wesley.

Carson, S. (Ed.). (1999). *Shaping the future: Physics in mathematical mood*. Bristol: Institute of Physics.

Chabay, R., & Sherwood, B. (2008). Computational physics in the introductory calculus-based course. *American Journal of Physics, 76*, 307–313.

Chalmers, A. (1999). *What is this thing called science?* London: Open University Press.

Christian, W., & Esquembre, F. (2007). Modeling physics with easy Java simulations. *Physics Teacher, 45*(8), 475–480.

Feynman, R. (1967). *The character of physical law*. New York: MIT Press.

Garcia, F., Gascón, J., Higueras, L., & Bosch, M. (2006). Mathematical modelling as a tool for the connection of school mathematics. *Zentralblatt für Didaktik der Mathematik, 38*, 226–246.

Hake, R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics, 66*, 64–74.

Halloun, I., & Hestenes, D. (1985). The initial knowledge state of college physics students. *American Journal of Physics, 53*, 1043–1048.

Handelsman, J., Ebert-May, D., Beichner, R., Bruns, P., Chang, A., DeHaan, R., Gentile, J., Lauffer, S., Stewart, J., Tilghmen, S., & Wood, W. (2005). Scientific teaching. *Science, 304*, 521–522.

Hestenes, D. (1987). Toward a modelling theory of physics instruction. *American Journal of Physics, 55*, 440–454.

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *Physics Teacher, 30*, 141–158.

High Performance Systems. (1997). *Stella (version 5) [Computer software]*. Hannover: High Performance Systems.

Keiner, L., & Burns, T. (2010). Interactive engagement: How much is enough? *Physics Teacher, 48*, 108–111.

Mazur, E. (1997). *Peer instruction: A user's manual*. Upper Saddle River: Prentice-Hall.

McDermott, L. (1991). Millikan lecture 1990: What we teach and what is learned–closing the gap. *American Journal of Physics, 59*, 301–315.

McDermott, L. (1997). *Physics by inquiry*. New York: Wiley.

McDermott, L., & Redish, E. (1999). Resource letter: PER-1: Physics education research. *American Journal of Physics, 67*, 755–767.

National Research Council. (1989). *Everybody counts*. Washington: National Academies Press.

Neves, R., Silva, J., & Teodoro, V. (2008). Improving the general physics university course with computational modelling. Poster presented at the *2008 Gordon Research Conference, Physics Research and Education: Computation and Computer-Based Instruction*, Bryant University, Smithfield.

Neves, R., Silva, J., & Teodoro, V. (2009). Computational modelling with Modellus: An enhancement vector for the general university physics course. In A. Bilsel & M. Garip (Eds.), *Frontiers in science education research* (pp. 461–470). Famagusta: Eastern Mediterranean University Press.

Ogborn, J. (1985). *Dynamic modelling system*. Harlow: Longman.

Ogborn, J. (1994). Modelling clay for computers. In B. Jennison & J. Ogborn (Eds.), *Wonder and delight, essays in science education in honour of the life and work of Eric Rogers 1902–1990* (pp. 103–114). Bristol: Institute of Physics Publishing.

Redish, E. (2004). A theoretical framework for physics education research: Modeling student thinking. In E. Redish & M. Vicentini (Eds.), *Proceedings of the International School of Physics: Enrico Fermi Course CLVI* (pp. 1–63). Bologna: Italian Physical Society.

Redish, E., & Wilson, J. (1993). Student programming in the introductory physics course: M.U.P.P.E.T. *American Journal of Physics, 61*, 222–232.

Schwartz, J. (2007). Models, simulations, and exploratory environments: A tentative taxonomy. In R. Lesh, E. Hamilton, & J. Kaput (Eds.), *Foundations for the future in mathematics education* (pp. 161–172). Mahwah: Lawrence Erlbaum Associates.

Slooten, O., van den Berg, E., & Ellermeijer, T. (Eds.). (2006). *Proceedings of the International Group on Research on Physics Education (GIREP) 2006 Conference: Modelling in Physics and Physics Education*. Amsterdam: European Physical Society.

Teodoro, V. (2002). *Modellus: Learning physics with mathematical modelling*. PhD thesis. Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Lisboa, Portugal.